DOCUMENT RESUME

ED 218 130 SE 038 240

AUTHOR Casstevens, Thomas W.; And Others
TITLE Exponential Models of Legislative Turnover. [and] The Dynamics of Political Mobilization, I: A Model of the Mobilization Process, II: Deductive Consequences and Empirical Application of the Model. Applications of Calculus to American Politics. [and] Public Support for Presidents. Applications of Algebra to American Politics. Modules and Monographs in Undergraduate Mathematics and Its Applications Project. UMAP Units 296-300.
INSTITUTION Education Development Center, Inc., Newton, Mass.
SPONS AGENCY National Science Foundation, Washington, D.C.
PUB DATE 79
GRANT SED-76-19615; SED-76-19615-A02
NOTE 119p.

EDRS PRICE MF01 Plus Postage. PC Not Available from EDRS.
DESCRIPTORS Answer Keys; *Calculus; *College Mathematics; Higher Education; Instructional Materials; Learning Modules; *Mathematical Applications; *Mathematical Models; *Political Science; Politics; *Problem Solving

ABSTRACT

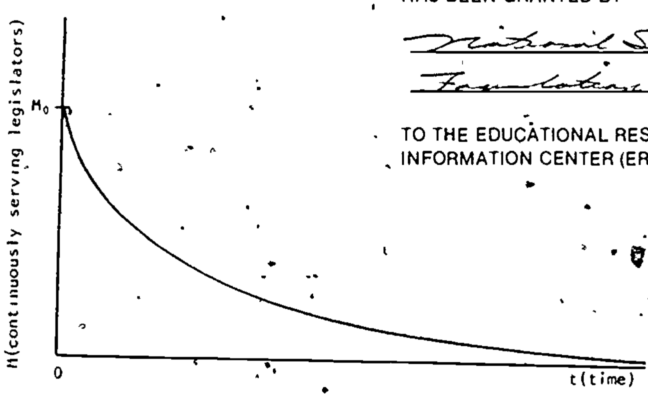
This document consists of five units which all view applications of mathematics to American politics. The first three view calculus applications, the last two deal with applications of algebra. The first module is geared to teach a student how to: 1) compute estimates of the value of the parameters in negative exponential models; and draw substantive conclusions about attrition processes from applications of negative exponential models. The next unit aims for pupils to gain an understanding of the role of recruitment and defection rates in political mobilization. The third module helps users understand some of the consequences and applications of a specific model. The last two modules make up a single section. Their aim is to enable the student to work with an elementary gain/loss model, and to understand some of the basic principles of the use of models to study political behavior. All the units contain exercises, and answers to all problem sets are provided. (MP)

umap

UNIT 296

EXPONENTIAL MODELS OF LEGISLATIVE TURNOVER

by Thomas W. Casstevens



APPLICATIONS OF CALCULUS TO AMERICAN POLITICS

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EXPONENTIAL MODELS OF LEGISLATIVE TURNOVER

by

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Intermodular Description Sheet: *UMAP Unit 296*

Title: EXPONENTIAL MODELS OF LEGISLATIVE TURNOVER

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Review Stage/Date: III 1/17/78

Classification: APPL CALC AMER POL/LEGIS TURNOVER

Suggested Support Material:

Prerequisite Skills:
1. An elementary calculus introduction to logarithmic and exponential functions.

Output Skills:
1. Be able to compute estimates of the value of the parameter in negative exponential models.
2. Be able to draw substantive conclusions about attrition processes from applications of negative exponential models.

Other Related Units:
The Dynamics of Political Mobilization I *(Unit 297)*
The Dynamics of Political Mobilization II *(Unit 298)*
Public Support for Presidents I *(Unit 299)*
Public Support for Presidents II *(Unit 300)*
Laws That Fail I *(Unit 301)*
Laws That Fail II *(Unit 302)*
Diffusion of Innovation in Family Planning *(Unit 303)*
Growth of Partisan Support I *(Unit 304)*
Growth of Partisan Support II *(Unit 305)*
Discretionary Support by Supreme Court I *(Unit 306)*
Discretionary Support by Supreme Court II *(Unit 307)*
What Do We Mean By Policy *(Unit 310)*

MODULES AND MONOGRAPHS IN UNDERGRADUATE
MATHEMATICS AND ITS APPLICATIONS PROJECT (UMAP)

The goal of UMAP is to develop, through a community of users and developers, a system of instructional modules in undergraduate mathematics and its applications which may be used to supplement existing courses and from which complete courses may eventually be built.

The Project is guided by a National Steering Committee of mathematicians, scientists and educators. UMAP is funded by a grant from the National Science Foundation to Education Development Center, Inc., a publicly supported, nonprofit corporation engaged in educational research in the U.S. and abroad.

This unit was presented in preliminary form at the Shambaugh Conference on Mathematics in Political Science Instruction held December, 1977 at the University of Iowa. The Shambaugh fund was established in memory of Benjamin F. Shambaugh who was the first and for forty years served as the chairman of the Department of Political Science at the University of Iowa. The funds bequeathed in his memory have permitted the department to sponsor a series of lectures and conferences on research and instructional topics. The Project would like to thank participants in the Shambaugh Conference for their reviews, and all others who assisted in the production of this unit.

This material was prepared with the support of National Science Foundation Grant No. SED76-19615. Recommendations expressed are those of the author and do not necessarily reflect the views of the NSF, nor of the National Steering Committee.

1. FIVE PROBLEMS

The turnover of legislators has considerable significance in theory and in practice: The possibility of electing new representatives is the essence of democracy in theory, and the prospect of replacing an incumbent stimulates ambition in practice. Scientists and politicians have made manifest efforts to measure or model, exploit or avoid such turnover.

We shall consider some problems related to the turnover and tenure of legislators. The problems arise, in practice, as inchoate desires: We wish (1) to forecast the future service of incumbent legislators, (2) to reconstruct past legislative service on the basis of fragmentary information, (3) to estimate the impact upon legislative service of a hypothetical event, (4) to measure an abnormal phenomenon, indirectly, by its impact upon legislative service, and (5) to compare legislative service in various legislative bodies. These problems have been stated as vague desires because such problems are not exactly formulated, at least initially, in practice.

Exact formulations of these problems are given in the examples and exercises, after the class of exponential models that is used to solve the problems.

2. THE EXPONENTIAL MODEL OF LEGISLATIVE TURNOVER

2.1 The Empirical Point of View

We view legislative service as an attrition process that begins at some specified time with a set of legislators and continues until some other time when those legislators have all ceased to serve. The process can be intuitively but precisely characterized as follows: Consider the members of a legislative body (briefly, a

legislature) after some election. Those legislators are the original members. With the occurrence of deaths, resignations, political defeats, etc., only some of the original members continue to be members after the next election. Those survivors are the re-elected members. With the occurrence of further deaths, resignations, political defeats, etc., only some of the re-elected members continue to be members after the next subsequent election. Those survivors are the re-re-elected members. This process can continue for an indefinite number of steps; but eventually, the continuous service of all original members is ended.

We assume that the rate of change in the number of continuously serving members is directly proportional to the number of continuously serving members or, in other words, that actual turnover is proportional to possible turnover. The plausibility of this assumption, as an empirical approximation, is suggested by the examples and exercises.

2.2 The Fundamental Equation

The assumption that the rate of change is constant is expressed by the differential equation

(1) $$\frac{dM}{dt} = -cM$$

where M is the number of continuously serving members at time t and c is a positive constant. The solution of this differential equation is

(2) $$M = M_0 e^{-ct}$$

where e is the irrational number 2.718... and M_0 is the number of original members.

Equation (2) is the fundamental equation in the exponential model. The characteristic appearance of this equation is displayed in Figure 1.

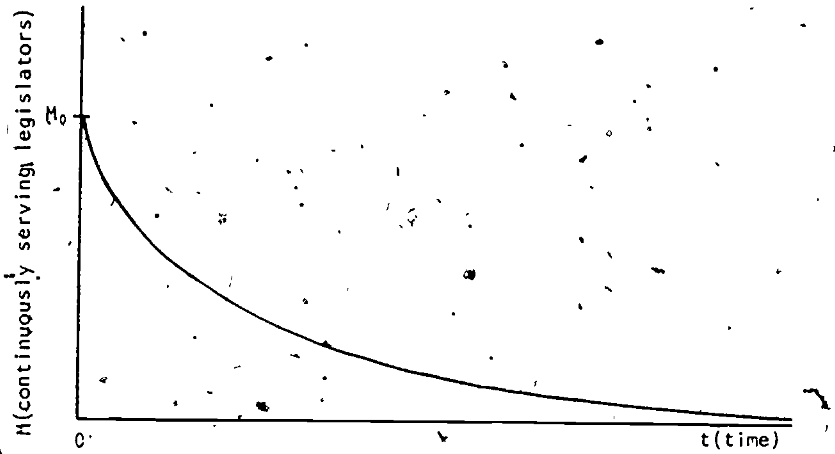

Figure 1. The number of continuously serving legislators decreases with time.

2.3 The Probability Interpretations

Equation (2) models the number of original members who serve continuously between t_0 and t, where t_0 is time zero for the process. The proportion of the original members, who serve continuously, is therefore

$$(3) \quad \frac{M_0 e^{-ct}}{M_0} = e^{-ct},$$

which is the probability that an original member serves continuously between t_0 and t. Since the original members either do or do not serve continuously,

$$(4) \quad P = 1 - e^{-ct}$$

is the probability that an original member's continuous service is ended by time t.

Equation (4) is the exponential probability distribution. The expectation of this distribution, which is the counterpart of the mean in discrete statistics, is equal to

$$(5) \quad \frac{1}{c},$$

so on the average the original members should continuously serve this long from t_0. The half-life of the distribution, which is the counterpart of the median in discrete

statistics, is approximately equal to

$$(6) \quad \frac{.693}{c}$$

so only half of the original members should continuously serve longer than this from t_0. These interpretations are invaluable in applications.

2.4 Estimating the Constant

Equations (1) through (4) are functions of time, but only the unit of measurement for time is vital for the exponential model of legislative turnover. The critical term in the model is the positive constant c whose value depends (in part) upon the unit of measurement for time.

The observed data, for a particular legislature, consist of the numbers of original members who continuously survive the elections between t_0 and t. Since the observations are only recorded around election time, the data are not continuous, although the exponential model itself is continuous. Figure 2 describes a typical case. The estimation problem is to find a value for c that generates an exponential curve that comes close to the observed numbers plotted in Figure 2.

A quick-and-dirty technique, for estimating the value of c, is based upon the half-life of the exponential distribution, as given in Equation (6). Consider the numbers that are graphed in Figure 2. There were 434 original members, and 214 continuously served for at least 8 years. We note that 217 is one-half of 434 and that the observed half-life should approximately equal the theoretical half-life. Since 214 is about 217, the theoretical half-life should be about 8 years. We set .693/c equal to 8 and solve for c, obtaining c = .0866 for the 1965 U.S. House of Representatives.

The standard technique, for estimating the value of c, is based upon the natural logarithms of the observed

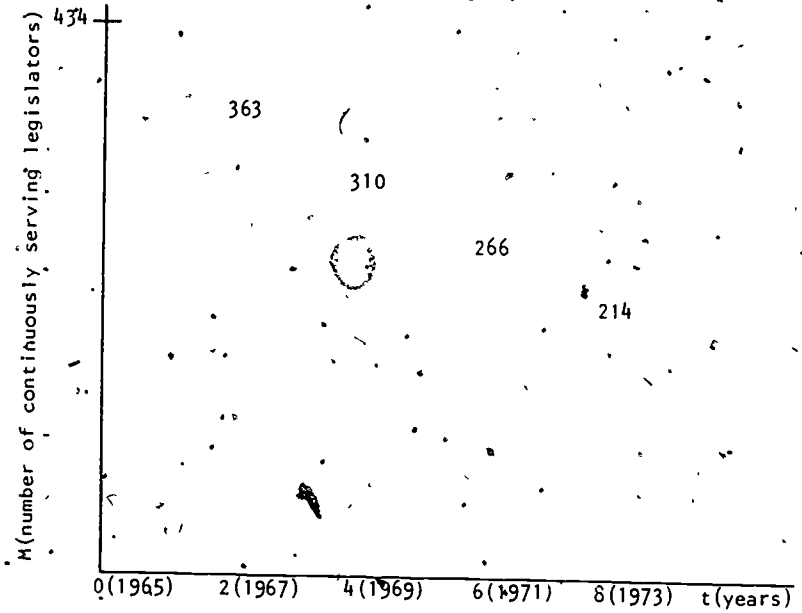

Figure 2. The numbers of continuously serving members after successive elections for the U.S. House of Representatives.

numbers of continuously serving members. Natural logarithms of exponentially distributed numbers fall on a straight line, with a slope of -c, since the natural logarithm function is the inverse of the exponential function. The value of c, for the straight line that best fits the natural logarithms of the observed numbers, is calculated using the formula

$$(7)\quad c = \frac{(\ln M_0)(t_1 + \dots + t_n) - [(\ln M_1)(t_1) + \dots + (\ln M_n)(t_n)]}{(t_1)^2 + (t_2)^2 + \dots + (t_n)^2},$$

where $(\ln M_i)$ is the natural logarithm of the i^{th} observed number of relevant members and t_i is the numerical value of the time (measured from t_0) of the i^{th} observed number. The best fitting straight line is implicitly defined by the criterion of ordinary least squares.

For an example of the standard technique, consider the numbers that are graphed in Figure 2. Their natural logarithms are approximately 6.0730, 5.8944, 5.7366, 5.5835 and 5.3660 for times in years of 0, 2, 4, 6 and 8 respectively. The sum of the times is equal to 20, and the sum of the squares of the times is equal to 120. We set c equal to [(6.0730)(20) - (5.8944)(2) - (5.7366)(4) - (5.5835)(6) - (5.3660)(8)]/120 and solve for c, obtaining c = .0858 for the 1965 U.S. House of Representatives. This estimate differs by .008 from the quick-and-dirty approximation.

Finally, with a computer, iterative techniques can be used to estimate the value of c by trapping and then finding the best value, where the best value is defined according to some criterion. These three techniques are illustrated in the examples.

3. FIVE EXAMPLES

3.1 The U.S. House of Representatives, 1965-1975

President Lyndon Johnson (Democrat) was returned to office by a massive majority in the presidential election of 1964. There was a concomitant landslide for his party's candidates for the House of Representatives. All Representatives took office in 1965. The numbers of continuously serving members, who surivived the subsequent four elections, are given in Figure 2.

We wished to forecast the number of continuously serving members of the 1965 House who would survive the election of November 1974. (This was actually done in a public lecture by the author in March 1974.) President Richard Nixon (Republican) was embroiled, at the time of our forecast, in the Watergate Scandal. Republican candidates were widely expected to have extraordinary difficulties in the upcoming election for the House.

Forecasts were calculated using estimated c values for the 1965 House and Equation (2), with $M_0 = 434$ and $t = 10$ (years) in this case. Three estimates of c were used: the quick-and-dirty approximation of .0866, the standard estimate of .0858, and an iterative estimate of .0853. The respective forecasts were $434e^{-.0866(10)} = 182.6$, $434e^{-.0858(10)} = 184.0$, and $434e^{-.0853(10)} = 184.9$. The actual number of survivors in the election was 174. Perhaps all three forecasts were surprisingly good, given the supposedly unusual character of the elections in 1964 and 1974. The quick-and-dirty approximation of c yielded the most accurate forecast, however, in this case.

3.2 The Andhra Pradesh Assembly, 1952-1967

The Assembly is the state legislature in Andhra Pradesh. There were state legislative elections in 1952, 1957, 1962, and 1967. Professor G. Ram Reddy and his associates made a detailed study of the 1967 Assembly. They reported (G. Ram Reddy, "Andhra Pradesh," in Iqbal Narain (ed.), State Politics in India, New Delhi, 1976.) that "about" sixty percent of the legislators were freshmen and that "nearly" eight percent had served continuously since 1952. We wish, on the basis of this fragmentary information, to estimate the unreported percentage who had served continuously since 1957.

The temporal perspective is reversed, when viewing continuous seniority as an attrition process, as shown in Figure 3. The percentage data are expressed as proportions in the graph for the Assembly. We guess, after inspecting the figure, that the half-life for the plotted data should be about 4 years. The quick-and-dirty technique, setting .693/c equal to 4 and solving for c, yields c = .173. The accuracy of this quick-and-dirty approximation is tested against the reported data. Equation (3) is the relevant formula for proportions of continuously serving members. We find that $e^{-.173(5)} = .421$ and $e^{-.173(15)} = .075$. These calculated proportions compare

favorably with the reported proportions of about .40 and nearly .08. Since a refined estimate of c can hardly be justified, given the fragmentary and approximate character of the observed data, the quick-and-dirty approximation is used to solve our problem: $e^{-.173(10)} = .177$, so about eighteen percent of the members should have served continuously since 1957.

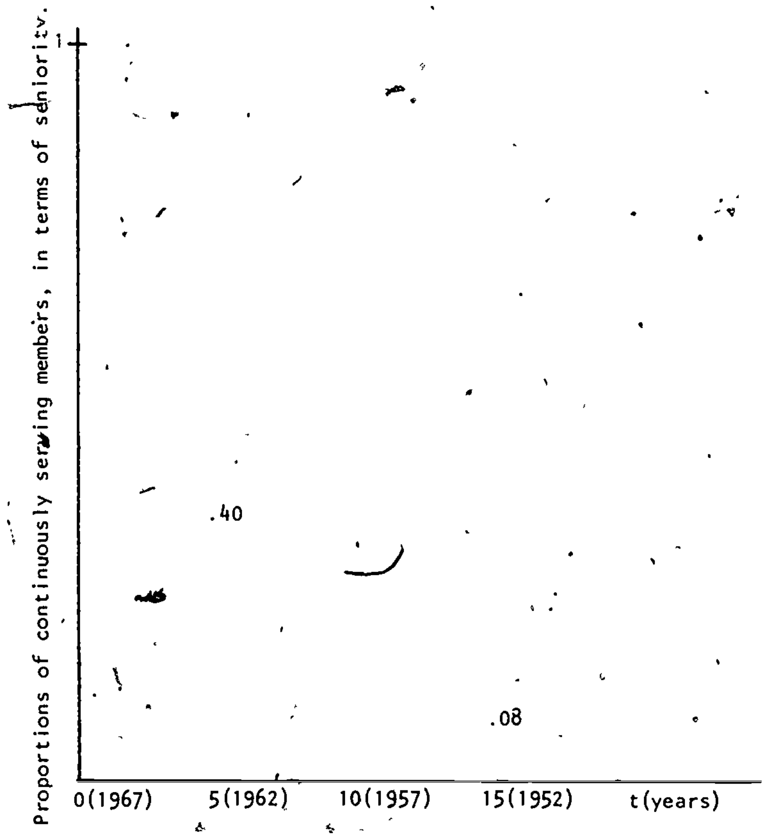

Figure 3. The proportions of continuously serving members are observed, looking backwards in time, after successive elections for the 1967 Andhra Pradesh Assembly.

Guessing and testing, as illustrated in this example, is often useful in applications of mathematics. (The singular verb is proper since guessing and testing is a unified method.) In particular, guessing and testing is indispensable for the discovery of mathematical models.

3.3 The British House of Commons, 1935-1940

The British House of Commons' life is limited to a maximum length of five years by the Parliament Act of 1911. Nevertheless, due to wartime conditions and by all party agreement, there was no general election between 1935 and 1945. The 1935 House had a life of ten years.

We wish to estimate what proportion of the original (1935) members would not have been re-elected if there had been a general election in 1940. Mr. Lawrence Murz (Oakland University, Department of Political Science, 1970), in his senior honors paper, estimated that $c = .130$, with time in years, for the 1935 House. (The estimate was made using the standard technique and was based upon continuous service from 1935 through 1970. Since British general elections were held at irregular times, time was measured in months in his original study.) With this estimate, the calculation is straightforward using Equation (4). The desired proportion is $1 - e^{-.130(5)} = 1 - .522 = .478$.

3.4 The Soviet Central Committee, 1956-1961

The Central Committee of the Communist Party promulgates authoritative policy decisions in the Soviet Union. Kremlinologists consider it to be roughly equivalent, in politics, to a unicameral legislature. The Central Committee is elected by the Party Congress. There were elections in February 1956, October 1961, March 1966, and March 1971.

First Secretary Nikita Khrushchev, in some semi-secret infighting, removed his opponents from the Central Committee in 1957. The number of members, who were

removed in this purge, has never been made public. We wish to estimate that number.

For 1956-1961 we assume that the total turnover was equal to normal turnover plus the purged members. The total turnover is a matter of public record. We estimate normal service with the exponential model. The 1956 Central Committee's full membership numbered 133; 66 were re-elected in 1961; 54 were re-re-elected in 1966; and 35 were re-re-re-elected in 1971. (See Thomas W. Casstevens and James R. Ozinga, "The Soviet Central Committee Since Stalin," American Journal of Political Science, Vol. 18, No. 3, (August 1974), pp. 559-568.) We note that 66 is about one-half of 133; but since the number 66 is itself assumed to be abnormal, the quick-and-dirty technique should not be used to estimate the value of c. We use the standard technique and, since the elections occurred at irregular times, measure time in months. The natural logarithms are approximately 4.8903, 4.1897, 3.9890, and 3.5553 for times 0, 68, 121, and 181 respectively. The sum of the times is equal to 370, and the sum of the squares of the times is equal to 52026. Equation (7) sets c equal to $[(4.8903)(370) - (4.1897)(68) - (3.9890)(121) - (3.5553)(181)]/52026$, yeilding $c = .0077$. The number of original members, who theoretically should have been re-elected, is then $133e^{-.0077(68)} = 78.8$, by Equation (3). We infer that actual turnover exceeded normal turnover by $78.8 - 66 = 12.8$ full members. This estimate of the size of the purge is a conservative estimate because normal service was itself calculated using the abnormally low figure for 1961. We conclude that at least one dozen full members were purged by Khrushchev.

3.5 The Central Committee and the House of Representatives, 1956 and 1965

We wish, in this example, to compare the 1956 Soviet

Central Committee and the 1965 U.S. House of Representatives. The values of the constant c, which represent the turnover rates, are very useful for this purpose. These values are estimated above, using the standard technique but differing units of time, as .0077 (U.S.S.R.) and .0858 (U.S.A.).

The units of time must be standardized for comparative purposes. Equation (2) holds, irrespective of the unit of measurement for time, for all exponential models of turnover in a given body of legislators. The relationship between the values of the constant and the units of measurement for time, in any two exponential models of a given legislature, is therefore

$$c_1 t_1 = c_2 t_2 \quad (8)$$

where time is measured, from the same starting point to the same instant, on different scales for model sub-one and model sub-two. In particular, for a given legislature, the value of the constant for a model in years is twelve times the value of the constant for a model in months.

We choose to standardize, in this example, in terms of years. The value of the constant thus becomes (.0077) (12) = .0924 for the Central Committee. The value of the constant remains .0858 for the House of Representatives. We note, as summary comparisons, that the expectation (1/c) is 10.8 years and 11.7 years and that the half-life (.693/c) is 7.5 years and 8.1 years, respectively. These figures suggest that the contemporary pattern of continuous legislative service, at the national level, is very similar in the Soviet Union and the United States.

4. EXERCISES

1. The 1965 U.S. House of Representatives.
   a. What is the value of the constant for time in months?
   b. How many continuously serving members should have been re-elected in the election of 1976?
   c. What proportion of the 1971 House of Representatives' 435 members should have had at least 6 years of continuous seniority?
2. The 1967 Andhra Pradesh Assembly.
   a. What is the value of the constant for time in months?
   b. What is the expectation for continuous seniority in years?
3. The 1935 British House of Commons, which had 615 original members, was elected in November 1935.
   a. What is the value of the constant for time in months?
   b. How many original members should have been re-elected in 1940?
   c. How many continuously serving members should have been re-elected in the election of October 1964?
   d. What is the expectation for continuous service in years?
   e. What is the half-life for continuous service in months?
4. The 1956 Soviet Central Committee.
   a. How many continuously serving full members should have been re-elected in the election of February-March 1976.
   b. What proportion of the 1971 Central Committee's 240 full members should have served continuously as full members since the election of 1956?
5. The 1957 Canadian House of Commons, which had 265 original members, was elected in June 1957. There were subsequent elections in March 1958, June 1962, April 1963, November 1965, and June 1968. The numbers of original members, who were successively re-elected, were 149, 87, 55, 42, and 23. (See Thomas W. Casstevens and William A. Denham III, "Turnover and Tenure in the Canadian House of Commons, 1867-1968," Canadian Journal of Political Science, Vol. 3, No. 4, (December, 1970), pp. 655-661.)
   a. Estimate the value of the constant for time in months, using the standard technique.

Prime Minister John Diefenbaker (Progressive Conservative) led his party's candidates to a landslide victory of unprecedented proportions in the election of March 1958.

b. How many original members should have been re-elected in the election of March 1958?

c. How many original members failed to be re-elected due to the landslide in 1958?

6. The 1953 U.S. Senate, at the beginning of the session, had an observed median continuous seniority of 6 years.

   a. Estimate the value of the constant for time in years, using the quick-and-dirty technique.

   b. What proportion of the members should have been serving continuously for at least 30 years?

7. Derive Equation (6) from Equation (4).

8. Derive Equation (8) from Equation (2).

5. ANSWERS TO EXERCISES

1. (a) .0072 if c = .0858 or .0866.

   (b) 155.0 if c = .0858; 153.5 if c = .0866. The actual number is not known by the author. The 1974 data might be included to re-estimate the value of the constant.

   (c) .65 if c = .0858 or .0866. The actual number was .61. [Note: The theoretical numbers of persons are given to one decimal place for two reasons: The numbers are theoretical. And a theoretical number such as 153.5 is exactly satisfied by an observation of 153 or 154.]

2. (a) .0144.

   (b) 5.8 years.

3. (a) .0108.

   (b) 323.1.

   (c) 14.6. Mr. Murz (op. cit.) reported that the actual number was 15.

   (d) 7.7 years.

   (e) 64.2 months.

4. (a) 21.0, using February. The actual number is not known by the author.

   (b) .14. Professors Casstevens and Ozinga (op. cit.) reported

that the actual proportion was 35 240 = .15.

5. (a) .019.

   (b) 223.4.

   (c) 74.4. This is a conservative estimate.

6. (a) .1155.

   (b) .08. The actual proportion was .01.

7. We set $1 - e^{-ct}$ equal to 1/2 and then solve for t in terms of c by taking the natural logarithm of each side of the equation: $e^{-ct} = 1/2$.

8. For two exponential models of a given legislative body, for the same time period but different time scales, we have

$$M_0 e^{-c_1 t_1} = M_0 e^{-c_2 t_2}$$

and after dividing by M_0, we obtain

$$e^{-c_1 t_1} = e^{-c_2 t_2}$$

which yields

$$-c_1 t_1 = -c_2 t_2$$

after taking the natural logarithm of each side, so that

$$c_1 t_1 = c_2 t_2$$

as desired.

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STUDENT FORM 1

Request for Help

Student: If you have trouble with a specific part of this unit, please fill out this form and take it to your instructor for assistance. The information you give will help the author to revise the unit.

Your Name________________________ Unit No.______

Page______
○ Upper
○ Middle
○ Lower

OR

Section______
Paragraph______

OR

Model Exam Problem No.______
Text Problem No.______

Description of Difficulty: (Please be specific)

Instructor: Please indicate your resolution of the difficulty in this box.

○ Corrected errors in materials. List corrections here:

○ Gave student better explanation, example, or procedure than in unit. Give brief outline of your addition here:

○ Assisted student in acquiring general learning and problem-solving skills (not using examples from this unit.)

Instructor's Signature________________________

Please use reverse if necessary.

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STUDENT FORM 2

Unit Questionnaire

Name.________________ Unit No.________________ Date________________

Institution________________ Course No.________________

Check the choice for each question that comes closest to your personal opinion.

1. How useful was the amount of detail in the unit?

____Not enough detail to understand the unit
____Unit would have been clearer with more detail
____Appropriate amount of detail
____Unit was occasionally too detailed, but this was not distracting
____Too much detail; I was often distracted

2. How helpful were the problem answers?

____Sample solutions were too brief; I could not do the intermediate steps
____Sufficient information was given to solve the problems
____Sample solutions were too detailed; I didn't need them

3. Except for fulfilling the prerequisites, how much did you use other sources (for example, instructor, friends, or other books) in order to understand the unit?

____A Lot ____Somewhat ____A Little ____Not at all

4. How long was this unit in comparison to the amount of time you generally spend on a lesson (lecture and homework assignment) in a typical math or science course?

____Much Longer ____Somewhat Longer ____About the Same ____Somewhat Shorter ____Much Shorter

5. Were any of the following parts of the unit confusing or distracting? (Check as many as apply.)

____Prerequisites
____Statement of skills and concepts (objectives)
____Paragraph headings
____Examples
____Special Assistance Supplement (if present)
____Other, please explain________________

6. Were any of the following parts of the unit particularly helpful? (Check as many as apply.)

____Prerequisites
____Statement of skills and concepts (objectives)
____Examples
____Problems
____Paragraph headings
____Table of Contents
____Special Assistance Supplement (if present)
____Other, please explain________________

Please describe anything in the unit that you did not particularly like.

Please describe anything that you found particularly helpful. (Please use the back of this sheet if you need more space.)

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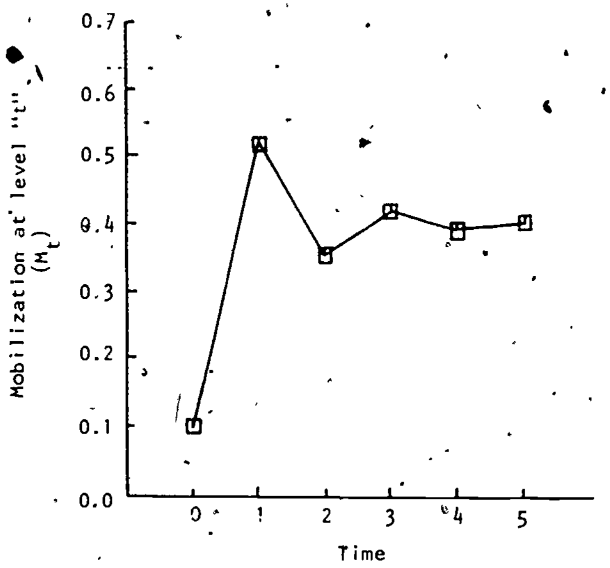
UNIT 297

THE DYNAMICS OF POLITICAL MOBILIZATION I: A MODEL OF THE MOBILIZATION PROCESS

by R. Robert Huckfeldt



APPLICATIONS OF CALCULUS TO AMERICAN POLITICS

THE DYNAMICS OF POLITICAL MOBILIZATION: I*

A Model of the Mobilization Process

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*This unit accompanies Unit 298, "The Dynamics of Political Mobilization II: Deductive Consequences and Empirical Application of the Model."

Intermodular Description Sheet: UMAP Unit 297

Title: THE DYNAMICS OF POLITICAL MOBILIZATION I: A MODEL OF THE MOBILIZATION PROCESS

Author: R. Robert Huckfeldt
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Notre Dame, Indiana 46556

Review Stage/Date: III 6/12/78

Classification: APPL CALC/AMER POL

Suggested Support Material:

References:

Boynton, G.R., "The American Revolution of the 1960's," (Paper presented at conference on mathematics and politics, Washington University, St Louis, June 15-18, 1975).

Goldberg, Samuel, Introduction to Difference Equations (New York Wiley, 1975).

Sprague, John, "Comments on Mobilization Processes Represented as Difference Equation Systems," (Washington University, St. Louis unpublished paper, 1976).

Prerequisite Skills

1. Knowledge of high school algebra.

Output Skills:

1. To gain an understanding of the role of recruitment and defection rates in political mobilization.

Other Related Units:

The Dynamics of Political Mobilization II (Unit 298)
Exponential Models of Legislative Turnover (Unit 296)
Public Support for Presidents I (Unit 299)
Public Support for Presidents II (Unit 300)
Laws that Fail I (Unit 301)
Laws that Fail II (Unit 302)
Diffusion of Innovation in Family Planning (Unit 303)
Growth of Partisan Support I (Unit 304)
Growth of Partisan Support II (Unit 305)
Discretionary Review by the Supreme Court I (Unit 306)
Discretionary Review by the Supreme Court II (Unit 307)
What Do We Mean by Policy? (Unit 310)

MODULES AND MONOGRAPHS IN UNDERGRADUATE MATHEMATICS AND ITS APPLICATIONS PROJECT (UMAP)

The goal of UMAP is to develop, through a community of users and developers, a system of instructional modules in undergraduate mathematics and its applications which may be used to supplement existing courses and from which complete courses may eventually be built.

The Project is guided by a National Steering Committee of mathematicians, scientists and educators. UMAP is funded by a grant from the National Science Foundation to Education Development Center, Inc., a publicly supported, nonprofit corporation engaged in educational research in the U.S. and abroad.

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This material was prepared with the support of National Science Foundation Grant No. SED76-19615. Recommendations expressed are those of the author and do not necessarily reflect the views of the NSF nor of the National Steering Committee.

1. INTRODUCTION

Many political events occur at a specific time in a particular context, but few political events are unrelated to the past and the future. Past political conditions shape the present just as present conditions have important implications for the future. Political conditions and events are usually part of political processes which can best be understood as occurring across time. Partisan political mobilization -- the enlistment of eligible participants in support of a political cause -- is such a phenomenon. The proportion of eligible participants who are mobilized is certainly a discrete event tied to a particular place and time, but the level of mobilization is dependent upon mobilization in the past and has implications for mobilization in the future.

This unit and Unit 298, The Dynamics of Political Mobilization: II, investigate the dynamic properties of political mobilization processes. Given limited information about a political mobilization process, what can be predicted regarding the outcome of the process? Can we predict whether levels of mobilization will be consistent or erratic from one time period to the next? Are the implications of similar mobilization processes different for political majorities and minorities? How is the mobilization process affected by the size of the pool of potential recruits?

Partisan political mobilization can refer to a variety of political behaviors: support for revolutionary political movements, participation in urban race riots, joining the Women's Christian Temperance Union, identification with a political party, or voting for a particular political candidate. In the discussions below, partisan mobilization refers to the percent of the eligible electorate voting for a particular party

in a given election. This convention, however, is primarily aimed at ease of discussion and does not limit the general nature of the substantive problem being investigated or the model being developed.

The present unit develops a simple model of the mobilization process and uses the model to simulate a number of different mobilization processes. Unit 298, The Dynamics of Political Mobilization: II, explores the model's deductive properties and applies it to an investigation of an actual mobilization process.

2. A MODEL OF THE MOBILIZATION PROCESS

A simplified model of the mobilization process is developed in this section to help answer the questions posed above. Before proceeding any farther, however, some terms and concepts must be precisely and arithmetically defined in symbolic notation.

2.1 Definitions

Individual actions performed within a spatial context determine the level of political mobilization. For example, suppose we are concerned with Democratic mobilization in Cook County, Illionis. A Cook County resident who votes for the Democratic Party is mobilized. The percent of Cook County residents who are mobilized is the level of mobilization in Cook County. All residents, however, are not eligible to participate in the mobilization process. Felons and people less than 18 years of age are not allowed to vote. Therefore, the level of mobilization is more accurately defined as the proportion of eligible participants who vote Democratic. In symbolic notation,

$$M = D/E.$$

Where:

M = the level of mobilization,
D = the number of Democratic voters,
E = the number of eligible participants.

The level of mobilization, however, is specific to a particular time. This time dependence can be symbolically expressed as:

$$M_t = D_t/E_t.$$

In more verbal terms, the mobilization level at time "t" is equal to the proportion of eligibles who are party supporters at time "t".

Changes in the level of mobilization can also be expressed in symbolic notation. A change operator -- "Δ" -- without superscript is defined to mean the change in the mobilization level from one time period to the next. Therefore, the following equality holds.

$$\Delta M_t = M_{t+1} - M_t.$$

The time sequence is a set of discrete, equally spaced points in time: t, t + 1, ..., t + n + 1. Each time point can be thought of as an election.

Finally, all eligible political participants are not susceptible to the recruitment efforts of all parties. While most small town Vermont bankers are legally eligible to vote, they are very unlikely to vote for the Democratic Party. An upper limit exists to the proportion of eligible participants which can be enlisted in support of any political cause. This limit is symbolically expressed as "L" and, in order to develop a more easily interpreted model, is assumed to be independent of time -- constant across time.

Exercise 1

The mobilization level is defined as the number of voters for a particular party divided by the number of eligible participants.

How does this definition substantively differ from one in which the number of voters for a particular party are divided by the number of voters for all the parties combined?

2.2 The Model

A simplified model of the mobilization process can be expressed with these symbols. Any change in the level of mobilization between "t" and "t + 1" is undoubtedly a function of two factors: (1) the rate at which individuals mobilized at "t" fail to continue their support at "t + 1" and (2) the recruitment rate among individuals who were not mobilized at "t" but are susceptible to a party's recruitment efforts. These two factors are symbolically expressed in the following model.

$$\Delta M_t = g(L - M_t) - f(M_t)$$

where:

g = the recruitment rate among those who are potentially eligible for recruitment but previously unmobilized
f = the defection rate among those who were previously mobilized.

Exercise 2

The model divides eligibles into three different categories. What are the categories? What other category might a more complex model include?

The model developed here is a difference equation. Difference equations are formal representations of ways in which quantities change over time. The quantity of interest here is the level of political mobilization.[1]

[1] For a more complete definition of difference equations see Goldberg (1958).

This difference equation can teach us several things concerning the properties of various mobilization processes. Two strategies exist for exploring the model's logical implications. Symbolic values can be specified and a sequence of mobilization levels generated upon those conditions. Alternatively, the symbolic values can be statistically estimated using data from actual mobilization processes. This unit explores the first alternative while Unit 298 explores the second.

3. SOME SIMULATED MOBILIZATION PROCESSES

Four values must be specified to generate a unique sequence of mobilization levels: "g", "f", "L", and "M_0" (assuming time -- "t" -- to be a series of consecutive integers beginning at zero). The difference between integers can be thought of as the time elapsing between equally spaced elections. Therefore, "M_0" is the initial mobilization level at the first election or the initial condition for the mobilization process being considered, "M_1" is the mobilization level at the next election, and so on.

3.1 Scenario One

In this first simulation, the party of interest is a majority party which has nearly exhausted its mobilization possibilities. The initial mobilization level -- "M_0" -- is .6 and the upper limit of people who might possibly be susceptible to the party's appeal is .7. So, while the party has only mobilized 60 percent of the eligible electorate, it has mobilized (.6/.7) or 86 percent of those people which it has any chance of mobilizing.

Furthermore, the party is losing old friends at a higher rate than it is making new ones. The rate of mobilization among unmobilized, potential supporters

is only .1 while the rate of losses among those who are mobilized is .3 (i.e., g = .1, f = .3). Therefore, the mobilization model can be rewritten in the following form:

$$\Delta M_t = .1(.7 - M_t) - .3(M_t)$$

where:

$$M_0 = .6.$$

Figure 1.1 shows the partial sequence of mobilization levels which is generated by this equation. The level of support for the party, especially in the early time periods, rapidly declines. The net rate of decline, however, comes

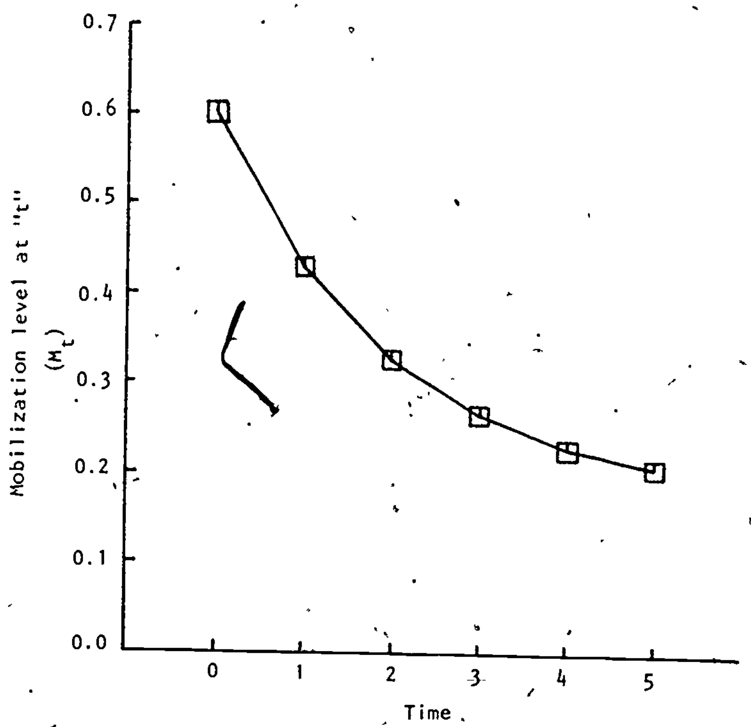

Figure 1.1. A partial sequence of mobilization levels for a party with the following parameters: g = .1, f = .3, and L = .7 (M_0 = .6).

closer to zero in each successive time period. As the process unfolds the levels of support decrease and, as a result, losses decline as well. In much the same fashion, the pool of unmobilized potential supporters increases allowing the same *rate* of gains to result in larger absolute increases in recruitment. In this way gains and losses come closer to offsetting each other.

Exercise 3

Put the model into a recursive form which can be used to generate a unique sequence of mobilization levels. That is, write the model in a way that expresses M_{t+1} as a function of M_t. (Remember: $\Delta M_t = M_{t+1} - M_t$.)

3.2 Scenario Two

Now consider a minority party which has not realized its potential. The party's level of support at the initial election being considered is only .2, but its limit of potential recruits is .7. Unlike the party previously considered, this party is experiencing a higher rate of gains than losses. Unmobilized potential recruits are enlisted at a .3 rate, while mobilized individuals defect from party ranks at a .1 rate. These values for the model's parameters result in the following equation:

$$\Delta M_t = .3(.7 - M_t) - .1(M_t)$$

where:

$$M_0 = .2.$$

The sequence of mobilization levels generated by this equation and shown in Figure 1.2 offers a contrast to the mobilization process previously considered. The party makes pronounced gains early in the time

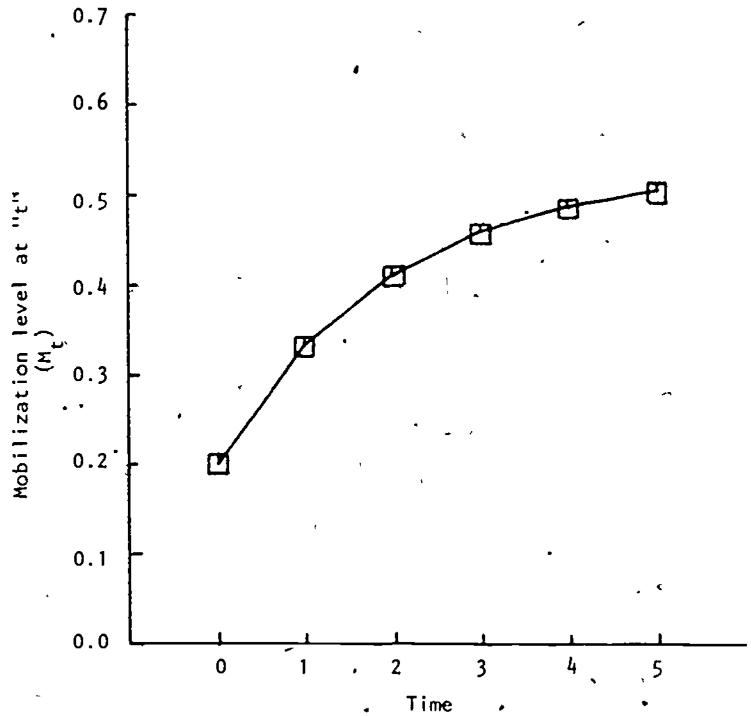

Figure 1.2. A partial sequence of mobilization levels for a party with the following model parameters: g = .3, f = .1, and L = .7 (M_0 = .2).

sequence, but these gains become less dramatic over time. As the party's mobilization level approaches the size of the pool from which it can gain new recruits, recruitment gains decrease. At the same time losses due to defection increase because the mobilized population has grown while the rate of defection has remained constant. As in the previous instance, the net changes in the mobilization level decrease over time.

3.3 Scenario Three

In the previous instances we considered a majority party which loses at a higher rate than it gains and a minority party which gains at a higher rate than it loses. Not surprisingly, the majority party declined in size and the minority party grew. It has also been demonstrated, however, that parties whose mobilization levels approach their pool of possible recruits are often hard pressed to continue growing. Imagine a majority party which gains converts at a higher rate that it loses old supporters. The party's initial level of support is .6, but the upper limit of the population which is susceptible to party recruitment efforts is only .7. The recruitment rate among unmobilized but potentially recruitable individuals is .3 while the defection rate among those previously recruited is .1. These conditions result in the following equation:

$$\Delta M_t = .3(.7 - M_t) - .1(M_t)$$

where:

$$M_0 = .6.$$

Even though the recruitment rate is larger than the defection rate, this majority party's support actually declines from its initial level in Figure 1.3. The change in mobilization is much less than the previous two instances, and the gradient of the change becomes even less pronounced as time passes. The scenario shows, however, that the direction of change is as much a function of initial mobilization levels with respect to recruitment limits as it is a function of recruitment and defection rates.

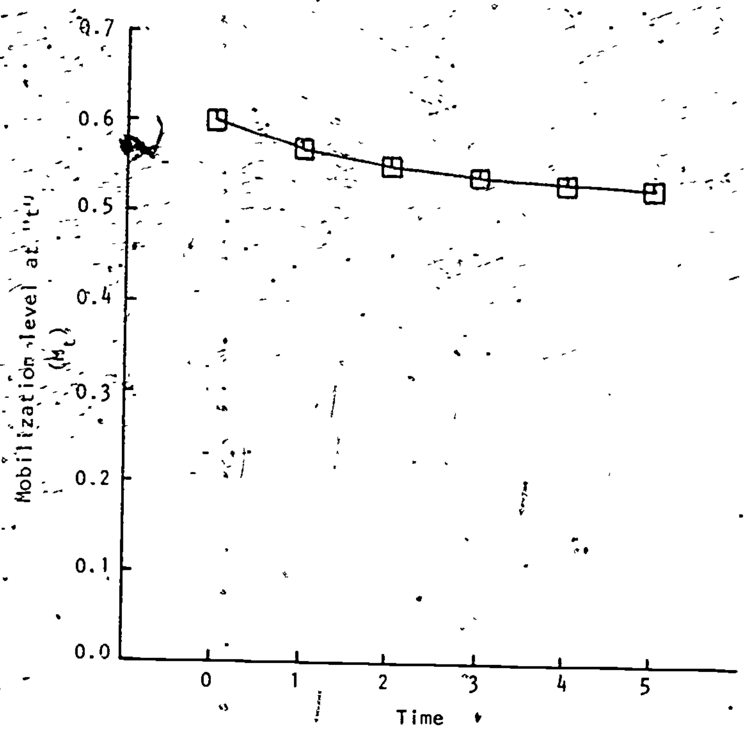

Figure 1.3. A partial sequence of mobilization levels for a party with the following parameters: g = .3, f = .1, and L = .7 (M_0 = .6).

Exercise 4

Could a party lose old converts at a higher rate than it gains new ones and still continue to grow?

Exercise 5

Specify recruitment and defection rates -- "g" and "f" -- for Scenario Three which would result in mobilization increases from the initial .6 level.

Exercise 6

What implications does the possible discrepancy between (1) rates of growth or decline in an absolute sense and (2) rates of recruitment and defection from subpopulations of eligibles have for the strategies of party leaders?

Exercise 7

In these first three simulations the size of the change has steadily decreased regardless of its direction. Do you think the direction of change would ultimately be reversed if the sequence was extended indefinitely?

3.4 Scenario Four

None of the mobilization processes considered thus far have involved extremely large turnover rates. The gain and loss parameters of the model have not exceeded .3. In this simulation imagine a more volatile political climate in which the turnover among both supporters and non-supporters is much higher. A majority party has an initial mobilization level of .6 and its ceiling of potential recruits is .8. The party's recruitment rate among those who are potentially subject to mobilization but previously unmobilized is .9. The defection rate among those who are already mobilized is .5. These conditions are summarized in the following equation.

$$\Delta M_t = .9(.8 - M_t) - .5(M_t)$$

where:

$$M_0 = .6.$$

The sequence of mobilization levels generated by this equation is shown in Figure 1.4. This sequence is significantly different from those previously considered. The direction of change in the other sequences was always monotonic: changes always occurred in the same direction.

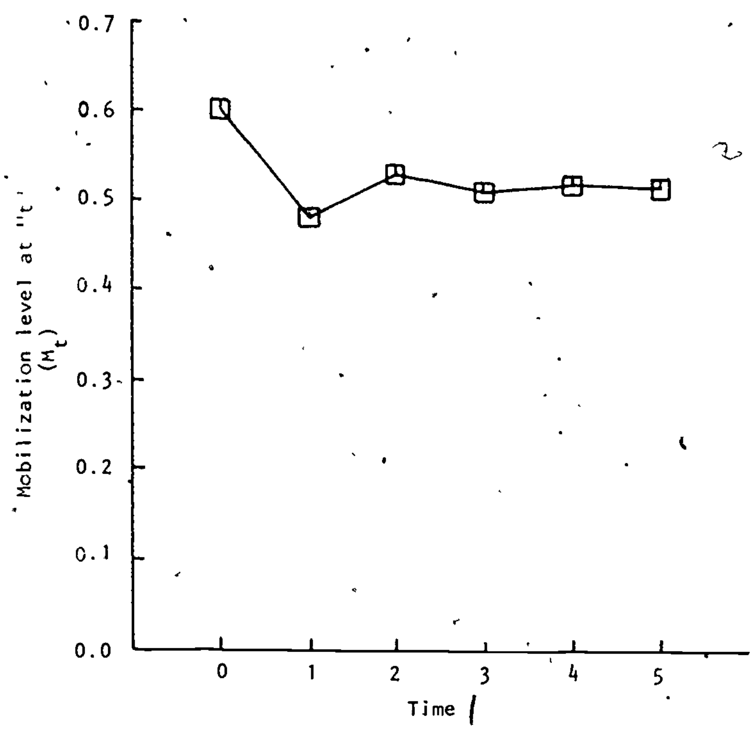

Figure 1.4. A partial sequence of mobilization levels for a party with the following parameters: g = .9, f = .5, and L = .8 (M_0 = .6).

The political party being considered either consistently lost or gained support even though the rate of absolute gains or losses varied. In this instance, losses and gains alternate. As in the previous instances, however, the absolute size of changes decreases in each succeeding time period. The process seems to settle down as time progresses.

3.5 Scenario Five

Finally, imagine a small party with a large growth potential which gains adherents at the same high rate that it loses previous converts. The party's initial mobilization level is only .1 but its limit of potential recruits is .8. The defection rate among previous supporters and the recruitment rate among previous non-supporters who are potentially eligible for mobilization are both .7. The conditions are shown in the following equation.

$$\Delta M_t = .7(.8 - M_t) - .7(M_t)$$

where.

$$M_0 = .1$$

The pattern of alternating gains and losses seen in Figure 1.4 is also present in the sequence of mobilization

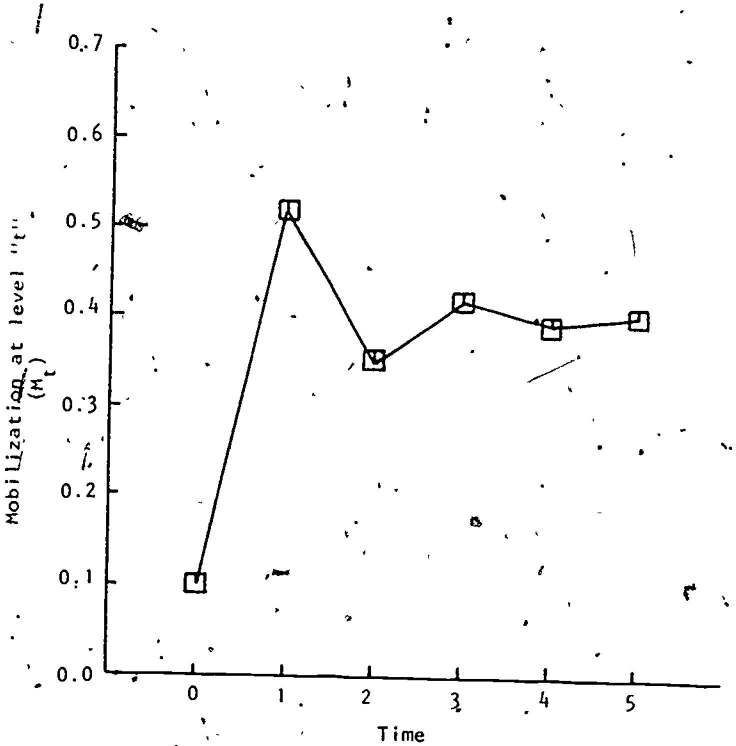

Figure 1.5. A partial sequence of mobilization levels for a party with the following parameters: $g = .7$, $f = .7$, and $L = .8$ ($M_0 = .1$)

levels generated by this equation and shown in Figure 1.5. The initial variation in mobilization levels is even more dramatic in this instance. Once again, however, the absolute value of the change decreases in each succeeding time period. Both of these latter two scenarios have involved very volatile political processes marked by both (1) a high turnover among party adherents and (2) fluctuating levels of overall support for the parties.

4. SUMMARY

This unit has demonstrated several properties of the mobilization process, as it is represented by our model, which are not intuitively obvious. All elements of the model - the three parameters as well as the initial mobilization level -- have important and inter-dependent consequences for the resulting mobilization process. No single parameter or subset of parameters can be used to typify a mobilization process. Furthermore, the same set of parameter values for "g", "f", and "L" can have very different implications depending upon the initial size of the party being considered.

Recruitment and defection rates ("g" and "f") mean different things to parties in different political circumstances. Parties which have more fully exploited their potential pool of recruits ("L") have a more difficult time achieving any additional growth. As Scenario Three illustrates, parties which recruit at a higher rate than they suffer defections can still decline in size.

The importance of recruitment and defection rates, however, is illustrated by comparing the first three scenarios with the last two. Changes in mobilization levels are monotonic in the first set of simulations regardless of the recruitment limits or initial mobilization levels. The parties either consistently increase or consistently decrease in size. Conversely, net gains alternate with net losses in the second set of simulations even though one simulation involves a minority and the other involves a majority party. In short, the high rates of defection and recruitment are related to the alternating increases and decreases in mobilization levels.

This unit's exploration strategy has been essentially inductive. A model is developed; some results are obtained, and some generalizations are drawn. Could we draw conclusions concerning what the sequence of mobilization levels for a given set of parameters would look like without generating the sequence? In other words, could we deduce the characteristics of a mobilization process from a knowledge of the parameters and the initial conditions? Unit 228, The Dynamics of Political Mobilization: II, explores the model's deductive properties as well as applying it to a consideration of an actual mobilization process.

5. ANSWERS TO EXERCISES

1. The base of all eligibles includes considerations regarding participation. This procedure measures the party's success at competing with apathy and the stay-at-home vote as well as its ability to compete with an opposing party or parties.

2. The three categories are (1) those already recruited: M_t, (2) those who are not supporters but might be: $L - M_t$, and (3) those not susceptible to party recruitment efforts $1 - L$. Another category could be those who would not, under any circumstances, defect from party ranks: H. The model would then become:

$$\Delta M_t = g(L - M_t) - f(M_t - H).$$

3. $M_{t+1} = M_t + g(L - M_t) - f(M_t) = (1 - g - f)M_t + gL.$

4. Yes. For example, consider a party with the following parameter values $L = .9$, $g = .1$, and $f = .4$. If the party's initial level of support is .1, its level of support at the next election would be .41

5. If M_{t+1} is equal to M_t then:

$$
\begin{aligned}
.6 &= (1 - g - f).6 + g(.7) \\
.6 &= .6 - .6g - .6f + .7g \\
0 &= -.6f + .1g \\
.6f &= .1g
\end{aligned}
$$

Therefore, in order for a party to grow from an initial mobilization level of .6, given that L equals .7, .1g must be greater than .6f.

6. A party's choice between allocating resources toward (1) recruiting new supporters or (2) insuring the continued support of those already recruited depends upon the relationship between the party's recruitment potential and its current level of support.

7. It would not.

STUDENT FORM 1

Request for Help

Return to:
EDC/UMAP
55 Chapel St.
Newton, MA 02160

Student: If you have trouble with a specific part of this unit, please fill out this form and take it to your instructor for assistance. The information you give will help the author to revise the unit.

Your Name________________________ Unit No.______

Page______ ○ Upper ○ Middle ○ Lower	OR	Section______ Paragraph______	OR	Model Exam Problem No.______ Text Problem No.______

Description of Difficulty: (Please be specific)

Instructor: Please indicate your resolution of the difficulty in this box.

○ Corrected errors in materials. List corrections here:

○ Gave student better explanation, example, or procedure than in unit. Give brief outline of your addition here:

○ Assisted student in acquiring general learning and problem-solving skills (not using examples from this unit.)

41

Instructor's Signature________________________

Please use reverse if necessary.

STUDENT FORM 2

Unit Questionnaire

Return to:
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Newton, MA 02160

Name______________________________ Unit No.____________ Date____________

Institution__________________________ Course No.______________________

Check the choice for each question that comes closest to your personal opinion.

1. How useful was the amount of detail in the unit?

____Not enough detail to understand the unit
____Unit would have been clearer with more detail
____Appropriate amount of detail
____Unit was occasionally too detailed, but this was not distracting
____Too much detail; I was often distracted

2. How helpful were the problem answers?

____Sample solutions were too brief; I could not do the intermediate steps
____Sufficient information was given to solve the problems
____Sample solutions were too detailed; I didn't need them

3. Except for fulfilling the prerequisites, how much did you use other sources (for example, instructor, friends, or other books) in order to understand the unit?

____A Lot ____Somewhat ____A Little ____Not at all

4. How long was this unit in comparison to the amount of time you generally spend on a lesson (lecture and homework assignment) in a typical math or science course?

Much	Somewhat	About	Somewhat	Much
____Longer	____Longer	____the Same	____Shorter	____Shorter

5. Were any of the following parts of the unit confusing or distracting? (Check as many as apply.)

____Prerequisites
____Statement of skills and concepts (objectives)
____Paragraph headings
____Examples
____Special Assistance Supplement (if present)
____Other, please explain______________________________

6. Were any of the following parts of the unit particularly helpful? (Check as many as apply.)

____Prerequisites
____Statement of skills and concepts (objectives)
____Examples
____Problems
____Paragraph headings
____Table of Contents
____Special Assistance Supplement (if present)
____Other, please explain______________________________

Please describe anything in the unit that you did not particularly like.

Please describe anything that you found particularly helpful. (Please use the back of this sheet if you need more space.)

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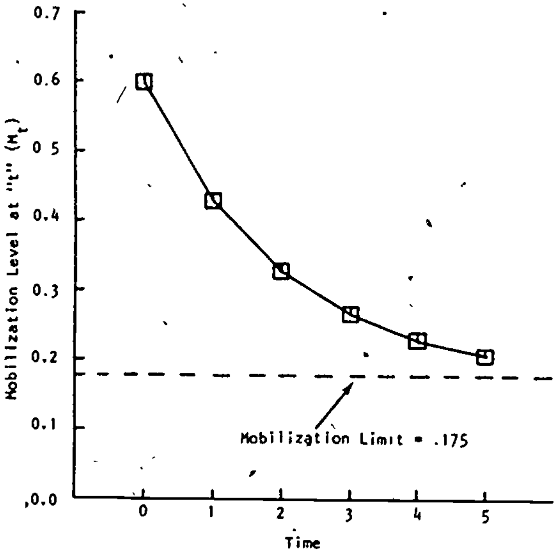
UNIT 298

THE DYNAMICS OF POLITICAL MOBILIZATION II: DEDUCTIVE CONSEQUENCES AND EMPIRICAL APPLICATION OF THE MODEL

by R. Robert Huckfeldt



APPLICATIONS OF CALCULUS TO AMERICAN POLITICS

THE DYNAMICS OF POLITICAL MOBILIZATION: II*

Deductive Consequences and Empirical Application of the Model

R. Robert Huckfeldt
Social Science Training and Research Laboratory
University of Notre Dame
Notre Dame, Indiana 46556

TABLE OF CONTENTS

*This unit accompanies Unit 297, "The Dynamics of Political Mobilization I: A Model of the Mobilization Process."

Intermodular Description Sheet: *UMAP Unit 298*

Title: THE DYNAMICS OF POLITICAL MOBILIZATION II: DEDUCTIVE CONSEQUENCES AND EMPIRICAL APPLICATION OF THE MODEL

Author: R. Robert Huckfeldt
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University of Notre Dame
Notre Dame, Indiana 46556

Review Stage/Date: III 6/12/78

Classification: APPL CALC/AMER POL

Suggested Support Material:

References:

Boynton, G.R., "The American Revolution of the 1960's," (Paper presented at conference on mathematics and politics, Washington University, St. Louis, Missouri, June 15-28, 1975).

Cadzow, James A., Discrete-Time Systems: An Introduction with Interdisciplinary Applications, Englewood Cliffs, New Jersey Prentice-Hall, 1973.

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Sprague, John, "Comments on Mobilization Processes Represented as Difference Equations of Difference Equation Systems," Washington University, St. Louis: unpublished paper, 1976.

Wonnacott, Thomas H. and Ronald J. Wonnacott, Introductory Statistics for Business and Economics, New York: Wiley, 1972.

Prerequisite Skills:

1. Knowledge of high school algebra.
2. Familiarity with The Dynamics of Political Mobilization I *(Unit 297)*

Output Skills:

1. To understand some of the consequences and applications of the model.

Other Related Units:

The Dynamics of Political Mobilization I *(Unit 297)*
Exponential Models of Legislative Turnover *(Unit 296)*
Public Support for Presidents I *(Unit 299)*
Public Support for Presidents II *(Unit 300)*
Laws that Fail I *(Unit 301)*
Laws that Fail II *(Unit 302)*
Diffusion of Innovation in Family Planning *(Unit 303)*
Growth of Partisan Support I *(Unit 304)*
Growth of Partisan Support II *(Unit 305)*
Discretionary Review by the Supreme Court I *(Unit 306)*
Discretionary Review by the Supreme Court II *(Unit 307)*
What Do We Mean by Policy? *(Unit 310)*

MODULES AND MONOGRAPHS IN UNDERGRADUATE MATHEMATICS AND ITS APPLICATIONS PROJECT (UMAP)

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This material was prepared with the support of National Science Foundation Grant No. SED76-19615. Recommendations expressed are those of the author and do not necessarily reflect the views of the NSF nor of the National Steering Committee.

1. INTRODUCTION

Unit 297, The Dynamics of Political Mobilization: I, developed a model of the mobilization process. Using that model, several sequences of mobilization levels were generated based upon different sets of simulated conditions. In this way various factors' effects upon the mobilization process were isolated and evaluated.

The present unit has two aims. First, a framework is developed with which to deduce the properties of a mobilization process based upon mathematical properties of difference equations. Second, the mobilization model is applied to the analysis of an actual rather than a simulated mobilization process.

2. THE MODEL'S DEDUCTIVE PROPERTIES

Expectations regarding the behavior and outcome of various political mobilization processes can be based upon model parameters and initial mobilization levels without inspecting the sequences of mobilization levels which are actually generated. This section develops the basis upon which these predictions are made. First, general and particular solutions to difference equations are defined and illustrated. A general solution is then developed for the difference equation which corresponds to the mobilization model. Finally, the model's deductive properties are outlined.

2.1 Solutions to Difference Equations

A difference equation solution is a single function which generates a sequence of values satisfying the equation at each time point. Consider the following simple case.

(1) $\Delta M_t = 2M_t$

or

(2) $M_{t+1} = M_t + 2M_t = 3M_t.$

One solution for this equation is $M_t = 3^t$. The solution results in the following equality based upon Equation (2).

(3) $3^{t+1} = 3(3^t).$

All the following solutions, however, also satisfy the equality: $2(3^t)$, $100(3^t)$, $.6(3^t)$. Each solution is a particular solution to the difference equation. A general solution, in contrast, provides a non-unique solution which is not related to any unique condition. All the particular solutions shown above are variants of the general solution -- $C(3^t)$ -- where C is any constant.

We make use of the following Theorem:[1] If: (1) a general solution is obtained for a linear difference equation of order "n" and (2) "n" consecutive values of the equation's generated sequence are defined, then it is the only solution to the difference equation with the prescribed conditions. To make use of this theorem, criteria must be established for the order and linearity of a difference equation. The order of a difference equation is defined to be the number of discrete intervals upon which the function depends. It is determined by subtracting the minimum time subscript from the maximum time subscript. In short, the mobilization model qualifies as a first order difference equation: $(t+1) - t = 1$. Furthermore, the model is linear because the coefficient for "M_t" is not a function of any "M_{t+k}".

[1] See Goldberg (1958).

Exercise 1

What is the order of each of the following difference equations?

(a) $X_t = a_2 X_{t-1} - a_1$

(b) $X_{t+2} = a_2 X_{t+1} + a_1$

(c) $X_{t+3} = a_2 X_t$

(d) $X_{t+2} = a_3 X_{t+1} + a_2 X_t + a_1$.

Exercise 2

Which of the following equations are linear? (Remember: Linear equations need not have constant coefficients.)

(a) $X_{t+1} = a_2 X_t^2$

(b) $X_{t+2} = a_2 X_t X_{t+1}$

(c) $X_{t+2} = a_2 t X_t + a_1$.

This theorem assures us that we can obtain a particular solution to any first order linear difference equation for which we know the general solution and any single sequential value for the function. Using the previous example where $M_{t+1} = 3M_t$, assume we know the value for M_0.

$$
\begin{aligned}
&M_1 = 3M_0 \\
&M_2 = 3M_1 = 3(3M_0) = 3^2 M_0 \\
(4)\quad &\cdot \\
&\cdot \\
&\cdot \\
&M_k = 3M_{k-1} = 3\left(3^{k-1}M_0\right) = 3^k M_0.
\end{aligned}
$$

In short, the particular solution is obtained by substituting M_0 for C.

2.2 Solving the Model

As you previously discovered in Exercise 3 of Unit 297, the mobilization model can be expressed in the following form:

$$(5)\qquad M_{t+1} = (1 - g - f)M_t + gL.$$

Since this equation is a first order linear difference equation, we only need to find a general solution and one sequential value for a given mobilization process to uniquely solve it.

Goldberg (1958) develops a solution for the following equation:

$$(6)\qquad X_{t+1} = a_1 + a_2 X_t.$$

This equation is the same form as the derived version of the mobilization model (Equation 5) where:

$$(7)\qquad a_1 = gL$$

$$(8)\qquad a_2 = (1 - g - f).$$

The solution can be found as follows:

$$
\begin{aligned}
&X_{t+1} = a_1 + a_2 X_t \\
&X_{t+2} = a_1 + a_2 X_{t+1} = a_1 + a_2(a_1 + a_2 X_t) = a_1(1 + a_2) + a_2^2 X_t \\
(9)\quad &\cdot \\
&\cdot \\
&\cdot \\
&X_{t+k} = a_1(1 + a_2 + \ldots + a_2^{k-1}) + a_2^k X_t.
\end{aligned}
$$

Exercise 3

Find the solution for X_{t+4}.

The quantity $(1 + a_2 + \ldots + a_2^{k+1})$ can be expressed in a more manageable closed form by summing a finite geometric series. First, set the quantity equal to "S".

$$S = (1 + a_2 + \dots + a_2^{k-1}). \tag{10}$$

Multiply both sides by a constant: "a_2".

$$a_2 S = (a_2 + a_2^2 + \dots + a_2^k). \tag{11}$$

Subtract Equation (11) from Equation (10).

$$S - a_2 S = (1 + a_2 + \dots + a_2^{k-1}) - (a_2 + a_2^2 + \dots + a_2^k) \tag{12}$$

or

$$S(1 - a_2) = (1 - a_2^k). \tag{13}$$

Finally, dividing both sides by $(1 - a_2)$ results in the closed form for the original Equation (10).

The difference equation solution can therefore be stated as follows:

$$X_{t+k} = a_2^k X_t + a_1\left[(1 - a_2^k) / (1 - a_2)\right] \quad \text{if } a_2 \neq 1 \tag{14}$$

$$X_{t+k} = X_t + k a_1 \quad \text{if } a_2 = 1. \tag{15}$$

Exercise 4

If we know the general solution and one sequential value we can obtain the particular solution. What if we know the sequence value for $t = 38$ instead of $t = 0$? How could we solve the equation for $t < 38$?

2.3 What Good Has This Done?

Now that we have a solution what can be done with it? Using the solution we can predict both (1) the *outcome* of a sequence and (2) the *behavior* of a sequence as it approaches the outcome. Several possible sequence behaviors and outcomes are considered here.[2]

[2] The discussion that follows is a non-rigorous treatment that depends heavily upon the discussion contained in Goldberg (1958).

A constant sequence. First, a difference equation can generate a sequence of equal values. In this case the outcome of the difference equation is the same as its initial value and the sequence's behavior is constant. Whenever the initial value of a sequence equals $(a_1/(1 - a_2))$ and "a_2" does not equal 1, the resulting sequence is constant. This can be shown using the Equation (14) solution.

$$X_{t+k} = a_2^k X_t + a_1((1 - a_2^k)/(1 - a_2))$$

$$X_{t+k} = a_2^k X_t + a_1/(1 - a_2) - a_2^k(a_1/(1 - a_2)) \tag{16}$$

$$X_{t+k} - a_1/(1 - a_2) = a_2^k\left[X_t - (a_1/(1 - a_2))\right].$$

So, if "X_t" (the initial value) equals $(a_1/(1 - a_2))$, the right hand side of Equation (16) is equal to 0 and "X_{t+k}" equals $(a_1/(1 - a_2))$ as well.

Some other sequence outcomes. Our consideration of other sequence outcomes is made simpler if we only consider the absolute value of difference equation sequences. The absolute value of sequences generated by difference equations can increase without bound or converge toward some limit as well as staying constant. It can be seen by inspecting the solution in Equation (14) that the absolute values for the sequence will continue to grow larger at an ever accelerating rate if "a_2" is greater than 1 or less than -1. Therefore, if "a_2" is greater than 1 or less than -1, the absolute value of "X_{t+k}" approaches infinity as "k" approaches infinity.

Rather than growing without bound, the absolute value of the difference equation sequence will converge toward a limit if either: $(-1 < a_2 < 0)$ or $(0 < a_2 < 1)$. Both the "$a_2^k X_t$" and "a_2^k" terms in the Equation (14) solution approach zero if either condition holds.

Therefore, the sequence generated by the difference equation approaches the limit:

(17) $a_1/(1 - a_2)$.

This value is subsequently expressed as "M*".

Some other sequence behaviors. What can be said regarding the behavior of a difference equation sequence as it approaches its outcome? Returning to the Equation (14) solution, "a_2^k" oscillates between negative and positive values if "a_2" is less than zero. Similarly, the sequence generated by the solution also oscillates regardless of the values for "X_t" or the solution's other term: $a_1((1 - a_2^k)/(1 - a_2))$. That is, declines in the mobilization level are followed by increases, and increases in the mobilization level are followed by declines.

Alternatively, "a_2^kX" grows or declines monotonically (constantly) whenever "a_2" is greater than zero. The difference equation sequence *declines* monotonically if "X_0" (the initial condition) is greater than "M*" and *increases* monotonically if "X_0" is less than "M*". Expectations regarding the outcome of a difference equation and the behavior of the sequence as it approaches the outcome are summarized in Figure 2.1.

Exercise 5

What can we predict about a difference equation function for which we know the general solution but not the particular solution? What can we not predict from the general solution alone?

2.4 The Expectations in Terms of the Model

These mathematical expectations can be expressed in notation applicable to the mobilization model. First,

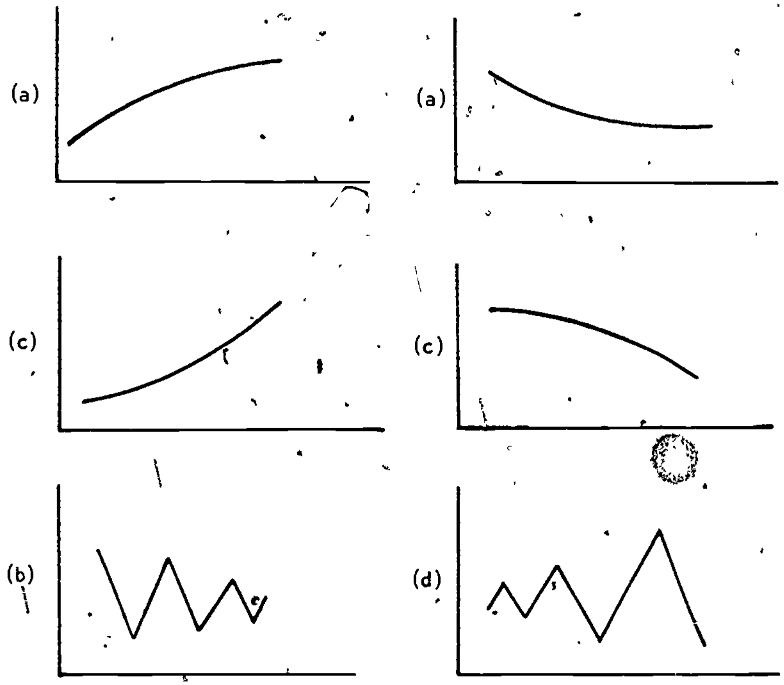

Figure 2.1 Expectations regarding the difference equation: $X_{t+1} = a_1 + a_2X_t$ (the initial condition cannot equal $(a_1/(1 - a_2))$.

		direction of a_2	
		$a_2 > 0$	$a_2 < 0$
absolute value of a_2	$\lvert a_2 \rvert < 1$	monotonic convergent (a)	oscillatory convergent (b)
	$\lvert a_2 \rvert > 1$	monotonic divergent (c)	oscillatory divergent (d)

consider the limit of the process: "M*". The limit is equal to $a_1/(1 - a_2)$, but, making use of the equalities in Equations (7) and (8), the limit can also be expressed as:

(18) $$M^* = gL/(g + f).$$

Verbally, the limit of the mobilization process is the ratio of (1) the recruitment rate multiplied times the upper limit of the population which is potentially susceptible to a party's recruitment efforts to (2) the sum of the recruitment and defection rates.

The "a_2" term provides an interesting and important analogy to the model (recall that: $a_2 = 1 - (g + f)$). The sum of "g" and "f" must be greater than 2 or less than -2 for a divergent sequence to result. Neither condition is possible by definition. If either parameter were negative, we would be dealing with positive losses or negative gains. Furthermore, neither parameter can be greater than 1; a party cannot lose more supporters than it already has or gain more than those that are eligible for conversion. These definitional contradictions in the model are related to an empirical impossibility. No party can gain or lose adherents indefinitely; unlimited growth cannot occur. In order for the model to be credible, "a_2" cannot be greater than 1 or less than -1.

The expectations developed here can be applied to the consideration of mobilization scenarios undertaken in the previous unit -- "The Dynamics of Political Mobilization: I." These applications are made in Table 2.1 and, for purposes of graphic display, the mobilization limit of Scenario One is presented in Figure 2.2.

Table 2.1

The Scenarios of Unit 297

	a_1	a_2	limit (M^*)	sequence behavior
Scenario 1	.07	.60	.175	monotonically decreases ($M_0 > .175$)
Scenario 2	.21	.60	.525	monotonically increases ($M_0 < .525$)
Scenario 3	.21	.60	.525	monotonically decreases ($M_0 > .525$)
Scenario 4	.72	-.40	.514	oscillatory convergent
Scenario 5	.56	-.40	.400	oscillatory convergent

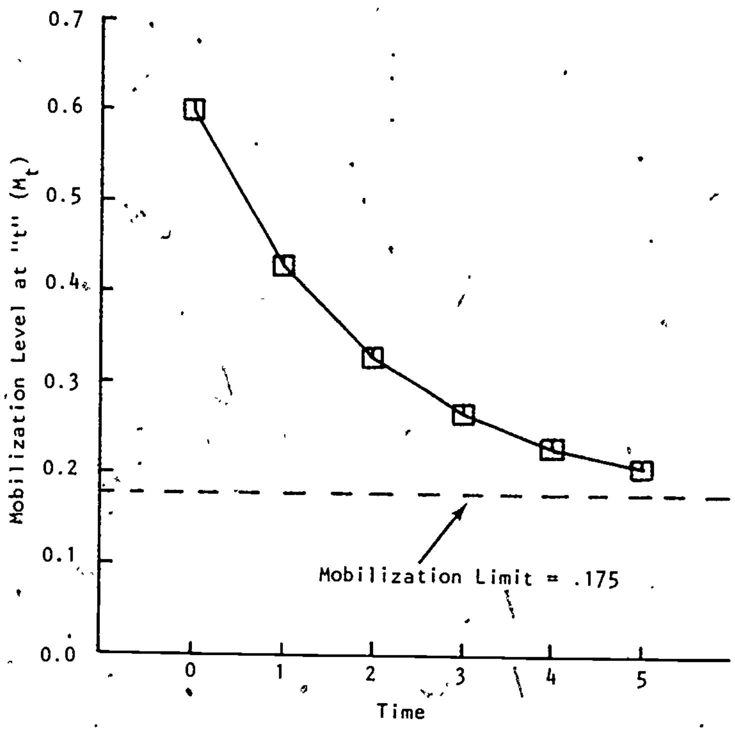

Figure 2.2 The mobilization limit and a partial sequence of mobilization levels for a Party with the following parameters: g = .1, f = .3, and L = .7 (M_0 = .6).

Exercise 6

What is the tipping point, in terms of "g" and "f", between an oscillatory and monotonic mobilization process?

3. DEMOCRATIC MOBILIZATION IN LAKE COUNTY, INDIANA

Previous considerations using the model have focused upon simulated mobilization processes. This section applies the model to an analysis of Democratic Party mobilization in Lake County, Indiana, from 1920 through 1968. This period is an important one in American politics which includes the return to normalcy following World War I, the Great Depression and the New Deal, the Eisenhower years, and the social turbulence of the 1960s. Lake County, which includes Gary, is an especially appropriate site for such an investigation since it has contained large concentrations of the population groups upon which Democratic ascendancy has been based: industrial workers, blacks, and the poor.

3.1 Statistical Estimation

The two coefficients -- "a_1" and "a_2" -- for the difference equation shown in Equation (6) can be statistically estimated on the basis of historic levels of Democratic mobilization in Lake County. The method used to estimate the coefficients is a statistical technique known as ordinary least squares (OLS). Given a Cartesian plane with a plot of data such as that shown in Figure 2.3, OLS fits a straight line with constant terms of "a_1" -- the intercept -- and "a_2" -- the slope. This OLS line provides the best fit to the data because it minimizes the sum of the squared discrepancies from the line. A single discrepancy or error is defined as the distance between an observed point in the plane and the

line, perpendicular to the horizontal axis (Wonnacott and Wonnacott, 1972).

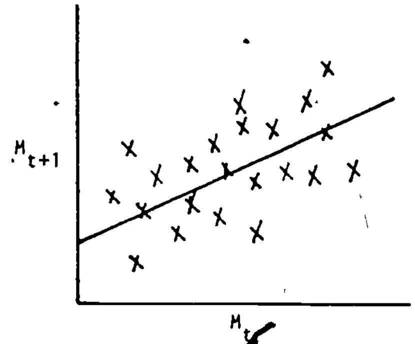

Figure 2.3 An example of an Ordinary Least Squares Regression Line.

Some hard to resolve statistical problems occur because we must statistically explain a given mobilization level in terms of a preceding mobilization level.[3] Our discussion ignores these problems; the scope of such a consideration would go beyond the bounds set here. This example is aimed at showing the application of the model to an actual mobilization process rather than producing accurate, unbiased coefficient estimates.

OLS was used to estimate the coefficients in the following model:

$$M_{t+1} = a_1 + a_2 M_t. \tag{19}$$

where:

M_t = the proportion of Lake County adults voting for the Democratic presidential candidate in year "t"

t = 1920, 1924, ..., 1964

t+1 = 1924, 1928, ..., 1968.

[3] See Hibbs (1974) for a consideration of these problems.

The resulting OLS estimates are .14 for "a_1" and .62 for "a_2".[4]

3.2 Estimating the Model Parameters

While calculating the values for "a_1" and "a_2" on the basis of "g", "f", and "L" was a simple task, the reverse is not so easily accomplished. As Equations (7) and (8) show, three unknown values must be defined on the basis of only two known values. This is an impossible undertaking unless an additional constraint can be imposed upon one of the three parameters. The model parameters, however, were chosen and defined to provide a descriptive representation of the mobilization process. Therefore, we can introduce some additional constraints upon the parameters in order to insure their descriptive adequacy.

Several reasonable restrictions can be imposed upon the three parameters. They are the following:

(20) $0 < L < 1$

(21) $0 < g < 1$

(22) $0 < f < 1$

The restriction contained in inequality (20) is based on the assertion that at least some subset of the adult population is potentially susceptible to party recruitment efforts, but the subset cannot equal or exceed the

---

[4] The Bureau of the Census issues population counts every ten years. Therefore, adult population estimates for elections occurring between census counts were derived using a simple technique of linear interpolation. For example, the 1924 estimate was derived as follows:

$$A_{1924} = A_{1920} + 4(A_{1930} - A_{1920})/10.$$

A_x symbolizes the number of adults living in Lake County during year "x". In this example an estimate for 1924 is derived from census figures for 1920 and 1930.

size of the adult population. The second and third restrictions (inequalities 21 and 22) are based on the previously discussed implausibility of negative losses and gains and the fact that losses and gains cannot exceed the size of the relevant populations. A negative loss would be a gain, but it is impossible to recruit those parts of the population which are already mobilized. Similarly, a negative gain would constitute a loss, but a party cannot lose supporters it does not already have. Finally, it would be impossible to lose or gain more than that part of the population which is eligible to be lost or gained. A perfect gain or loss rate of either zero or one might be conceivable, but the possibility is sufficiently remote to justify the restrictions.

Equation (7) can easily be rearranged to result in $(a_1/g + L)$. Therefore, using the right side of inequality (20),

(23) $a_1/g < 1$ or $a_1 < g$.

Slight manipulation of Equation (8) results in $(f = 1 - g - a_2)$. Substituting this equality into the left side of inequality (17) produces:

(24) $0 < 1 - g - a_2$ or $g < 1 - a_2$.

So, "g" lies in the interval bounded by "a_1" and "$1 - a_2$" which is shown in the number line representation of Figure 2.4. Lacking better information it is

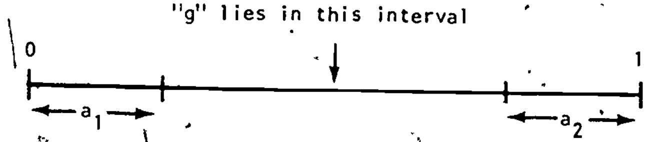

Figure 2.4 Interval within which the estimate for "g" lies ($a_1 < g < 1 - a_2$).

reasonable to suppose that "g" lies in the middle of the interval. This assumption results in the following constraint:

$$g = a_1 + ((1 - a_1 - a_2)/2). \tag{25}$$

The assumption that "g" lies in the *middle* of the interval bounded by "a_1" and "$1 - a_2$" is more than a blind guess. If we assume that a normal distribution of estimates exists within the interval, then the probability of choosing an accurate estimate for "g" is enhanced by picking the midpoint (see the bell-shaped probability distribution of Figure 2.5). Multiple

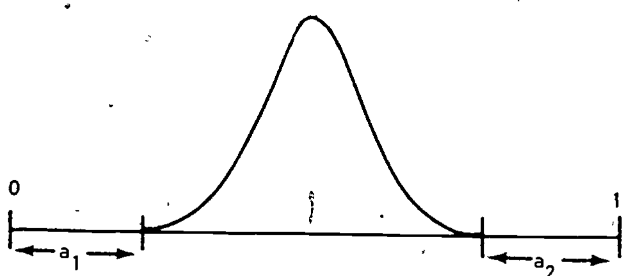

Figure 2.5 Probability distribution for the estimates of "g", assuming that the estimates for the parameter are normally distributed within the interval.

estimates exist for "g" and for the other parameters because our choice of time points and elections is only a sampling from a universe of mobilization levels. For example, we can choose off-year or presidential-year elections, and we have a variety of elections to choose from for a given series of years. Therefore, the resulting parameters are estimates of the "true" underlying parameters.

In most instances the interval bounded by "a_1" and "$1 - a_2$" will be small making the assumption a

fairly safe one. We should, however, be sensitive to two potential problems. First, "a_1" must obviously be less than "$1 - a_2$". Second, if "a_2" is negative, "$1 - a_2$" is greater than one. This means that the interval may be significantly larger, depending upon "a_1", than if "a_2" was positive.

Exercise 7

Using the estimates for Erie County, what interval does "g" lie within?

The three model parameters can be estimated using the following system of equations and the OLS estimates for "a_1" and "a_2".

$$\begin{aligned} g &= a_1 + ((1 - a_1 - a_2)/2) \\ f &= 1 - a_2 - g \\ L &= a_1/g. \end{aligned} \tag{26}$$

3.3 Applying the Model to Lake County

Based on this system of equations, the following Lake County estimates are obtained for the parameters of the mobilization model.

$$\begin{aligned} g &= .26 \\ f &= .12 \\ L &= .54. \end{aligned} \tag{27}$$

The parameter estimates suggest that the Democrats' recruitment rate has been over twice as large as their defection rate. Only slightly more than half of the population, however, appears susceptible to party recruitment efforts.

Two Democratic mobilization paths are shown for Lake County in Figure 2.6; the observed sequence of

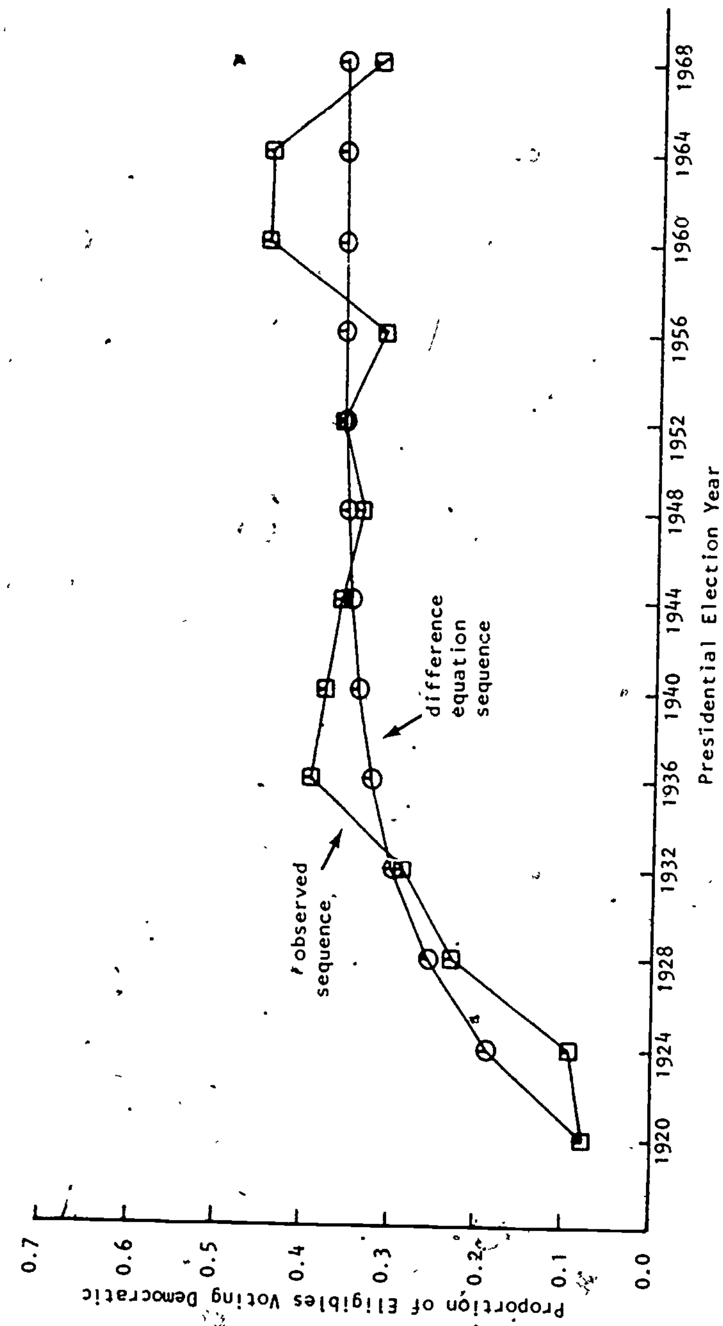

Figure 2.6 The observed and difference equation generated paths of Democratic mobilization in Lake County Presidential elections from 1920 through 1968.

$a_1 = .14$ $g = .26$

$a_2 = .62$ $f = .12$

$M^* = .37$ $L = .54$

mobilization levels and the difference equation path generated with 1920 as the initial mobilization level and the above parameter estimates. The difference equation representation of the process is a monotone increasing sequence which converges toward a limit (M*) of .37. There are, however, observed mobilization levels which lie above the limit. This fact is not troublesome if we (1) view the model parameters as being constant factors operating throughout the period and (2) treat discrepancies from the difference equation path as deviations due to irregular factors not included in the model. For example, the path based upon the same model parameters with the 1960 mobilization level as its initial condition converges toward the same limit, but it would be a monotone decreasing sequence.

At first inspection these mobilization levels may appear somewhat low. Only 54 percent of the population is susceptible to the Democratic Party's recruitment efforts and the mobilization process converges toward a level where only 37 percent of the population is mobilized in support of the Democratic Party. The level of mobilization, however, is defined to the base of all eligibles rather than all voters. Therefore, these mobilization levels reflect overall turnout as well as partisan support. The mean Lake County turnout rate for presidential elections from 1956 to 1968 was approximately 69 percent. Using this 69 percent figure as a norm, the pool of potential Democratic supporters is 78 percent the size of the average turnout. The "M*" or limit of Democratic support is 54 percent of the average turnout. In short, rather than indicating weakness these two estimates -- "L" and "M*" -- give witness to the *strength* of the Democratic Party in Lake County.

Exercise 8

Since we did not constrain the *statistical* estimation of "a_2" what would you have concluded if the estimate for "a_2" had been greater than 1 or less than -1?

4. SUMMARY

This unit, and Unit 297, The Dynamics of Political Mobilization: I, have shown several things. Political events can profitably be viewed as being interdependent across time. The past is related to the present, and both are therefore related to the future. In particular, political mobilization is a process rather than a series of discrete events.

Predictions can be made regarding the outcome and behavior of a political mobilization process on the basis of a simple mathematical model. The limit of the process can be determined, and the oscillatory or monotonic progress of the path can be specified. The model can be used to simulate mobilization processes or to analyze processes which have occurred in the past.

Finally, similar mobilization processes have different consequences for political parties of different sizes. Two parties with the same recruitment limits and recruitment and defection rates also have the same mobilization limits regardless of their initial mobilization levels. Therefore, the same processes can result in a net gain for one party and a net loss for another.

5. ANSWERS TO EXERCISES

1. (a) first
   (b) first
   (c) third
   (d) second
2. (c)
3. $X_{t+4} = a_1(1 + a_2 + a_2^2 + a_2^3) + a_2^4 X_t$
4. Yes. If $(X_{t+1} = a_1 + a_2 X_t)$, then $(X_t = -a_1/a_2 + 1/a_2\, X_{t+1})$. Therefore, the difference equation must be solved in reverse direction where: t = 38, 37, 36, ... 1, 0.
5. We can predict (1) whether the sequence has a limit or an equilibrium, (2) what the limit is, and (3) how the sequence will approach the outcome, i.e., divergence or convergence. We cannot predict whether a monotone sequence will be increasing or decreasing.
6. A sequence is oscillatory whenever the sum of (g + f) is greater than one.
7. $.14 < g < .38$.
8. One appropriate conclusion would be that the model provides implausible results. That is, for these data and this mobilization process, the model is inadequate.

STUDENT FORM 1

Request for Help

Return to:
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Student: If you have trouble with a specific part of this unit, please fill out this form and take it to your instructor for assistance. The information you give will help the author to revise the unit.

Your Name ______________________ Unit No. ______

Page ______
○ Upper
○ Middle
○ Lower

OR

Section ______
Paragraph ______

OR

Model Exam Problem No. ______
Text Problem No. ______

Description of Difficulty: (Please be specific)

Instructor: Please indicate your resolution of the difficulty in this box.

○ Corrected errors in materials. List corrections here:

○ Gave student better explanation, example, or procedure than in unit. Give brief outline of your addition here:

○ Assisted student in acquiring general learning and problem-solving skills (not using examples from this unit.)

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STUDENT FORM 2

Unit Questionnaire

Name________________________ Unit No.____________ Date____________

Institution________________________ Course No.____________________

Check the choice for each question that comes closest to your personal opinion.

1. How useful was the amount of detail in the unit?

____Not enough detail to understand the unit
____Unit would have been clearer with more detail
____Appropriate amount of detail
____Unit was occasionally too detailed, but this was not distracting
____Too much detail; I was often distracted

2. How helpful were the problem answers?

____Sample solutions were too brief; I could not do the intermediate steps
____Sufficient information was given to solve the problems
____Sample solutions were too detailed; I didn't need them

3. Except for fulfilling the prerequisites, how much did you use other sources (for example, instructor, friends, or other books) in order to understand the unit?

____A Lot ____Somewhat ____A Little ____Not at all

4. How long was this unit in comparison to the amount of time you generally spend on a lesson (lecture and homework assignment) in a typical math or science course?

Much	Somewhat	About	Somewhat	Much
____Longer	____Longer	____the Same	____Shorter	____Shorter

5. Were any of the following parts of the unit confusing or distracting? (Check as many as apply.)

____Prerequisites
____Statement of skills and concepts (objectives)
____Paragraph headings
____Examples
____Special Assistance Supplement (if present)
____Other, please explain________________________________

6. Were any of the following parts of the unit particularly helpful? (Check as many as apply.)

____Prerequisites
____Statement of skills and concepts (objectives)
____Examples
____Problems
____Paragraph headings
____Table of Contents
____Special Assistance Supplement (if present)
____Other, please explain________________________________

Please describe anything in the unit that you did not particularly like.

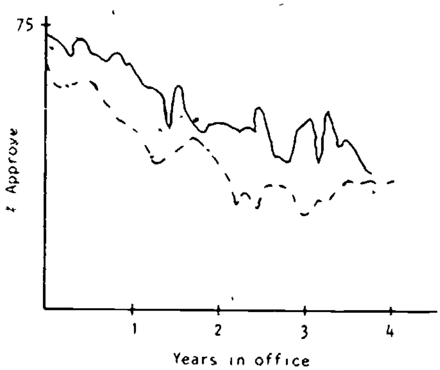
Please describe anything that you found particularly helpful. (Please use the back of this sheet if you need more space.)

UMAP MODULE

MODULES AND
MONOGRAPHS IN
UNDERGRADUATE
MATHEMATICS
AND ITS
APPLICATIONS

Public Support for Presidents

by Barbara Salert



Applications of Algebra to American Politics

Birkhäuser Boston Inc.
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PUBLIC SUPPORT FOR PRESIDENTS

by

Barbara Salert
Department of Political Science
Washington University
St. Louis, Missouri 63130

TABLE OF CONTENTS

Intermodular Description Sheet: UMAP Units 299 and 300

Title: PUBLIC SUPPORT FOR PRESIDENTS

Author: Barbara Salert
Department of Political Science
Washington University
St. Louis, Missouri 63130

Review Stage/Date: IV 8/30/79

Classification: ALGEBRA/AMER POL

Suggested Support Materials (helpful, but not necessary):

Unit 299: Graph paper and a calculator.
Unit 300: Graph paper, and access to a computer or programmable calculator.

Prerequisite Skills:

1. High-school algebra.
2. Knowledge of elementary probability (helpful, but not necessary).
3. Section 2.3 may require prior or concurrent instruction in regression analysis for full comprehension.

Output Skills:

1. Be able to work with an elementary gain/loss model, and understand some of the basic principles of the use of models to study political behavior.

Other Related Units:

Exponential Models of Legislative Turnover *(Unit 296)*
The Dynamics of Political Mobilization I *(Unit 297)*
The Dynamics of Political Mobilization II *(Unit 298)*
Diffusion of Innovation in Family Planning *(Unit 303)*
Growth of Partisan Support I *(Unit 304)*
Growth of Partisan Support II *(Unit 305)*
Discretionary Review by Supreme Court I *(Unit 306)*
Discretionary Review by Supreme Court II *(Unit 307)*

This material was prepared with the partial support of National Science Foundation Grant No. SED76-19615 A02. Recommendations expressed are those of the author and do not necessarily reflect the views of the NSF or the copyright holder.

1. THE MODEL

1.1 Introduction

Popular support for a president is often taken as a convenient, if somewhat crude, indication of the fate of an administration and some of its major policies. For one thing, presidents who are facing a second election cannot afford to incur too much public wrath. Such presidents are therefore likely to exhibit some sensitivity to poll results. But the president is not the only one who is concerned about the results of popularity polls. Public support for a president also seems to carry over to popular opinion about the administration and the political party of the president. The president's party, for example, does considerably better in congressional elections if the president has managed to gain a large amount of public support at the time of the election. Thus, congressmen facing re-election and potential future congressmen are also concerned about a president's success in winning popular support.

President Nixon's popularity ratings, prior to his resignation in 1974, illustrate the relevance of public opinion for analyses of presidential politics. Table 1 clearly indicates that Nixon's popularity plummeted throughout 1973. By the beginning of 1974, his ratings had tapered off to a low of about 25% of the population approving of the way Nixon handled the presidency. By the time of his resignation, then, Nixon had moved from being a highly popular president to being a most unpopular one.

The fate of Nixon's administration is, of course, an extreme example. Most presidents do not suffer a scandal of Watergate proportions, nor do most resign from office. Nevertheless, Nixon's decline in popularity

TABLE 1

Trend in Nixon's Popularity, 1973-1974

	Approve	Disapprove
January, 1973	68%	25%
February	65	25
March	59	32
April	48	40
May	44	45
June	45	45
July	40	49
August	38	54
September	32	59
October	27	60
November	27	63
December	29	60
January, 1974	26	64
February	25	64
March	26	65
April	26	65
May	28	61
June	26	61

Source: Gallup Opinion Index

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is not as unusual as it might appear at first glance. Presidents typically find that their public support decreases throughout the course of their term in office. Figure 1 illustrates this phenomenon for some recent presidents. Clearly, Nixon's fate at the polls was not peculiar; in fact, presidents are generally unable to maintain the kind of popular support they enjoyed at the beginning of their presidential term.

From one point of view, this phenomenon is quite surprising. After all, different presidents pursue

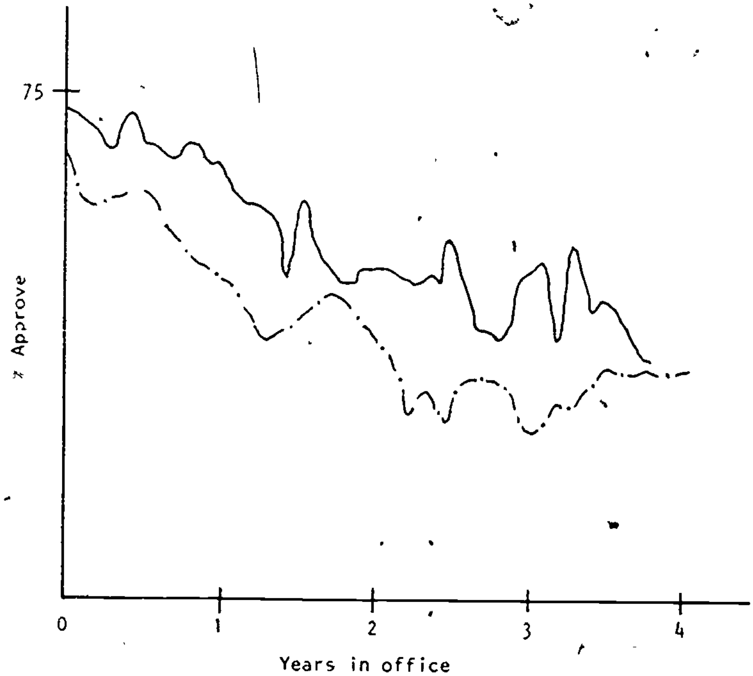

Figure 1. Trends in Presidential Popularity

Source: Gallup Opinion Index

Key: ———————— Johnson, January 1965 - August 1968
— — · — · — Truman, January 1949 - November 1952

---

different policies and these policies appeal to some people and alienate others. The notion that presidents deliberately pursue policies that offend increasingly large segments of the population seems absurd in a country where politicians depend on public support to gain access to their office. But if presidents do not systematically pursue policies that offend increasing numbers of people, why does popular support for a president decline over time?

A possible explanation links this phenomenon to the nature of U.S. electoral politics.[1] Prior to an election, presidents campaign vigorously as they seek to mobilize support for their candidacy. However, once in office, a president has far less time and opportunity to engage in the extraordinary kinds of mobilization efforts that are typical of campaign periods. It is therefore possible that a president will begin each term with an unusually high level of support. After all, in most cases a victorious president has managed to win a majority of the popular vote. And, in the general excitement following the election, the president may well pick up some additional good will from others in the country.

As the excitement of a campaign dies down, and politics take on a more normal aspect, the commitment to a particular president that was elicited during the campaign and its aftermath probably weakens for many people. The president will lose some supporters if he pursues policies that are disagreeable to them or is generally unable to maintain the kind of economic and political conditions his supporters expect to obtain. On the other hand, the president is likely to gain some supporters from people who benefit from his policies or who simply find that things are considerably better than they expected them to be. The balance between loss of support and gain of new adherents will determine changes in a president's popularity over time. Of course, there is no necessary reason why this balance should wind up on the negative side

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1 For further discussion of this issue see John E. Mueller, Wars, Presidents and Public Opinion (New York: John Wiley, 1973). The model presented here was originally developed by John Sprague and used to study public opionon by G. R. Boynton, "Sources of Change in Confidence and Trust in Government," paper presented at the 1974 Annual Meeting of the American Political Science Association, Chicago, Illinois, August 29 - September 2, 1974.

for most presidents. But if it is true that some of the initial support a president receives is artificially high, in the sense that it stems from campaign performances or even from an initial extraordinary effort to win public support, then it is likely that as the campaign dies down the president will find himself losing more supporters than he gains adherents.

The phenomenon of decreasing presidential support, then, may be explicable without positing the existence of callous presidents who fail to maintain support because they are totally contemptuous of public opinion. But if it is true that the loss of presidential popularity can be attributed to the unusually high support levels generated by campaigns which subsequently decrease to more normal levels that are determined by the general attitudes of the population, presidential policies, and existing political and economic conditions, then several questions remain to be answered. For one thing, while it may be plausible to suppose that in a two party system initial support for a president will be unusually high, it is also plausible to suppose that not all presidents will suffer a loss of support or, at least, that not all presidents will suffer a loss of support to the same degree. What, then, determines how much support a president loses during his term in office? Can presidents ever gain support over the course of their incumbency? If so, under what conditions? Then, too, there is the question of what constitutes "normal" support levels. If presidential support decreases to some normal level, what determines how high this level will be? How fast do the effects of the campaign wear off so that this level is approached? Do all presidents have some normal support level, or do some generate such controversy that their support fluctuates wildly over time? If support can fluctuate wildly, under what circumstances would it be likely to do so? Are these circumstances likely to occur in the context of American politics?

A simple model of presidential popularity may be helpful in answering these and other questions. The model that is presented in Section 2 is undoubtedly a highly simplified representation of the realities of American politics. As such, it cannot hope to capture all of the complexities involved in public opinion about an incumbent president. Nevertheless, in many circumstances the model provides a close enough approximation to actual conditions that it can help us understand why, in a democratic political process, so many presidents generate a trend of increasing political disaffection with their administration.

1.2 A Model of Presidential Popularity

A simple model of the way in which support for a president changes over time can be formulated by noting that there are only two ways in which the level of support can possibly change: 1) people who had previously supported the president withdraw their support or 2) people who had previously not supported the president change to a position of support for the president. Thus, if we know the level of support for a president at any time, we will know the level of support he will receive the next time his popularity is measured if we know how many of his previous supporters withdrew their support and how many new supporters he acquired in the interim period.

To formalize these ideas, let us suppose that a president's popularity is measured in equally spaced time intervals. Thus, we might have weekly or monthly or bimonthly information about the proportion of adults who currently approve of the way a given president is handling his job. This proportion will be represented as S_t, and called the *level of support at a given time* t. Thus, the support level is given by the formula

$$S_t = \frac{\text{adults who approve of the president at time } t}{\text{total adult population at time } t}.$$

Since the proportion of adults supporting the president is measured in equally spaced time intervals, we can represent successive time periods by successive non-negative integers. For example, if monthly data on presidential popularity ratings were available for January through December, we could treat January, the initial time for which data are available as $t = 0$. February would then correspond to the time $t = 1$, March would be represented by $t = 2$, and December by $t = 11$. The proportion of adults who support the president in July, in this example, would be S_6.

Change in the level of support accorded a president between any two successive time periods can be represented as ΔS_t. Thus,

$$\Delta S_t = S_{t+1} - S_t. \tag{2.1}$$

For example, using the monthly data from January to December outlined in the previous paragraph, $\Delta S_8 = S_9 - S_8$ would be the change in presidential support levels between September and October.

Exercise 1

Suppose presidential popularity were measured in weekly intervals between January 1 and December 31, with $t = 0$ for the first week in January. Assume 4 weeks to the month. How could you represent

a) the results obtained in the third week of January?
b) the results obtained in the first week in March?
c) the difference in popularity between the second and third weeks in February?

With this notation in mind, we can begin to formalize the ideas introduced at the beginning of this section. Since S_t represents the proportion of adults who support the president at t, the quantity $(1 - S_t)$ will represent the proportion of adults who do not support the president at t, either because they disapprove of him or because they have no opinions or are indifferent on the issue. The reason for measuring S_t as a *proportion* of the adult population supporting a president, rather than the absolute number of people supporting the president, is that the proportion measure allows an easy alternative interpretation of S_t. It is the average probability that an adult will support a given president at time t, in the following sense. If $S_1 = 0.6$ (60% of adults approve of the president) we might say that the average probability that an adult supported the president at the first time period was 0.6. Similarly, the average probability that an adult did not support the president in this instance would be 0.4.

Change in the average probability that an adult will support the president depends on the balance between the probability that the president loses support and the probability that the president gains support. Let f represent the probability that a person who supports the president will withdraw his or her support by the next time period. The amount of support the president loses, however, also depends on the amount of support he has. If there is a 10% chance that the president will lose a supporter between two successive time periods, and if he is supported by 90% of the population at the first time period, then the total loss of support he can expect to receive at the second time period will be $(0.1)(0.9) = 0.09$. That is, the president would only retain a 0.81 level of support if he gained no new adherents. If, on the other hand, the probability that a supporter defects is 0.1, but only one half of the population supports the president, the loss of support, in the absence of gains of new adherents, would be $(0.1)(0.5) = 0.05$. The probability of loss from time t to time t + 1 is the probability of loss given that someone is a supporter at time t, times the probability that someone is a supporter at time t. The quantity fS_t is the proportion of adults who withdraw their support from one time period to the next, a measure which can be interpreted as the probability of loss from time t to time t + 1.

Similarly, let g be the average probability that a person who does not support the president at some time will switch his or her view to support the president by the next time period measured. The quantity g is thus the rate of gain in support enjoyed by a president. The probability of gaining new adherents is given by the quantity $g(1 - S_t)$; alternatively, $g(1 - S_t)$ represents the proportionate gain in supporters between two successive time periods.

Since change in the support for a president depends on the balance between losses of and gains in support, a simple equation for the changes in presidential popularity that occur over the course of a presidential term can now be formulated:

(2.2) $$\Delta S_t = -fS_t + g(1 - S_t).$$

This equation simply states that changes in the level of support for a president between two successive time periods can be found by subtracting losses of support (the proportion of adults who withdraw support) from gains in support (the proportion of previous non-supporters who begin to support the president). Note, though, that the probability f that a supporter withdraws support and the probability g that a non-supporter becomes a new adherent are assumed to remain constant throughout the course of any presidential term. That is, neither f nor g is treated as a variable that changes over time. If we have monthly data for some president's term in office on presidential popularity ratings, the model assumes that both the loss rate and the gain rate will be the same for any two successive months in the term.

Clearly, the assumption that f and g are constant in any given presidential term is an oversimplified representation of reality. A president who ends an unpopular war might expect a large gain in support at the time he takes this action. If the press discovers that a president has taken bribes from major corporations, the president could expect unusually high losses of support when the information was made public. The probability that a supporter withdraws support or that a non-supporter begins to offer support, in short, depends on what is happening at the time, and social, political, and economic conditions that affect these probabilities do change over time.

Nevertheless, the examples mentioned above are highly unusual. Presidents rarely end unpopular wars or get caught taking bribes. Political life is usually considerably duller than that. In most periods, some people are hurt by changing political and economic circumstances, others are helped by these circumstances, and most are not noticeably affected. While some variation in loss and gain rates occur during a presidential term, the huge changes that could be expected as a result of ending an unpopular war or being caught in a major scandal are probably rare. Thus, even though the assumption of a constant loss rate and a constant gain rate is violated in real life, there are probably many times for which this assumption approximates political realities. The model, in short, will probably not be able to predict all of the variations in support ratings that occur during a presidential term because it fails to incorporate all of the factors that cause short term changes in losses and gains of support for a president. However, if these short-run sources of variation are minor, as compared with longer-run tendencies, the model should be able to explain and predict general trends in presidential popularity.

A second assumption that is implicit in the model can be modified to provide a better approximation to political realities. Notice that the loss rate, f, operates only on current supporters -- a representation that is eminently reasonable since no president can lose more support than he has. On the other hand, the gain rate, g, operates on all non-supporters. That is, everyone who does not support the president is viewed as a potential future supporter. This assumption is probably less reasonable. Table 2, which gives the range of support received by some recent presidents, indicates that no president is likely to ever have 100% of the population supporting him. At least, no recent president has managed to win as much as a 90%

TABLE 2

The Range of Presidential Popularity

President	Range of % of Approval Received
Ford	37% - 71%
Nixon	24% - 68%
Johnson	35% - 80%
Kennedy	57% - 83%
Truman	23% - 87%

Source: Gallup Opinion Surveys

approval rate. This is not surprising. There are undoubtedly some people who will never support a president because they are unshakable supporters of the opposition party, or because they are far more radical in their political views than any president would be, or simply because they refuse to support any president on idiosyncratic grounds. For most presidents, then, the pool of potential supporters will be less than 100% of the population.

These considerations can be incorporated into the model of presidential popularity by introducing an upper limit L to the proportion of adults who might potentially approve of a president. This can be done by assuming that the proportion of adults available to switch their support to the president at time t is not $(1 - S_t)$ -- the full proportion of non-supporting adults -- but only $(L - S_t)$ for some $L < 1$. Thus, instead of writing

$$(2.2) \qquad \Delta S_t = -fS_t + g(1 - S_t)$$

we write

$$(2.3) \qquad \Delta S_t = -fS_t + g(L - S_t).$$

This modification leaves us with a model that is still a very simple representation of the process generating political support for a president during his term in office. However, the simplicity of the model is somewhat deceiving. Simple as it is, the model is still powerful enough to generate plausible answers to many of the substantive questions about declining presidential support levels that were raised at the end of the last section. Using the model to generate these answers, though, requires some mathematical manipulation of the model to extract the various consequences that follow from the assumptions embodied in the model. The preliminary manipulation required for further analysis is presented in Section 3.

1.3 Change in the Level of Support over Time

The model of changes in public support for presidents that was developed in the last section allows us to calculate the levels of public support for a president, once the gain rate g, the loss rate f, the upper limit L on support, and the initial level of support S_0 for a president are known. To see this, recall that the model states that changes in popular support follow the law

$$\Delta S_t = -fS_t + g(L - S_t). \tag{3.1}$$

Suppose we know the initial level of support, S_0, as well as the values of f, g, and L. Then we can calculate the level of support the president will receive at the following time period, S_1:

$$\Delta S_0 = S_1 - S_0 = -fS_0 + g(L - S_0), \tag{3.2}$$

$$S_1 = S_0 - fS_0 + g(L - S_0). \tag{3.3}$$

After some simple algebraic manipulation, (3.3) becomes

$$S_1 = (1 - f - g)S_0 + gL. \tag{3.4}$$

If the values of S_1 can be calculated, then so can the value of S_2. For,

$$\Delta S_1 = S_2 - S_1 = -fS_1 + g(L - S_1); \tag{3.5}$$

$$S_2 = S_1 - fS_1 + g(L - S_1),$$

$$S_2 = (1 - f - g)S_1 + gL. \tag{3.6}$$

When f, g, L, and S_1 are known, it is a simple matter to compute the value of S_2.

In a similar fashion, it is easy to show that

$$S_3 = (1 - f - g)S_2 + gL. \tag{3.7}$$

And, in general,

$$S_{t+1} = (1 - f - g)S_t + gL. \tag{3.8}$$

Thus, if we know the values of the parameters f, g, and L, and the initial level of support, S_0, all subsequent levels of support can be generated by simple calculation.

Exercise 2

Suppose you are given the following parameter values: f = 0.05, g = 0.15, L = 0.8. Calculate the values of S_1 through S_{10} for an initial condition $S_0 = 0.8$.

(People without calculators will find it easier to get approximate results by rounding to two significant figures at each step.

Graph your results. What is happening to support levels?

1.4 The Impact of the Initial Level of Support

We can even develop a formula that will allow us to calculate the level of support at any time, S_t, from the value of the initial support level and the values of the parameters f, g, and L. To see this, note that since

$$S_1 = (1 - f - g)S_0 + gL \tag{4.1}$$

and

$$S_2 = (1 - f - g)S_1 + gL, \tag{4.2}$$

then by substituting Equation 4.1 into Equation 4.2, we have

$$S_2 = (1 - f - g)[(1 - f - g)S_0 + gL] + gL.$$

After rearranging terms,

(4.3) $$S_2 = (1 - f - g)^2 S_0 + gL[1 + (1 - f - g)].$$

Similarly, since

(4.4) $$S_3 = (1 - f - g)S_2 + gL,$$

we can calculate S_3 directly from S_0 by substituting Equation 4.3 into Equation 4.4. Specifically,

$$S_3 = (1 - f - g)\{(1 - f - g)^2 S_0 + gL[1 + (1 - f - g)]\} + gL.$$

And, after rearranging terms,

(4.5) $$S_3 = (1 - f - g)^3 S_0 + gL[1 + (1 - f - g) + (1 - f - g)^2].$$

If we perform the same operations to calculate S_4 in terms of S_0 and the parameters f, g, and L, we have:

(4.6) $$S_4 = (1 - f - g)^4 S_0 + gL[1 + (1 - f - g) + (1 - f - g)^2 + (1 - f - g)^3].$$

And, by extending this out as many time periods as are desired you can see that in general

(4.7) $$S_t = (1 - f - g)^t S_0 + gL[1 + (1 - f - g) + (1 - f - g)^2 + \ldots + (1 - f - g)^{t-1}].$$

While this formula is somewhat cumbersome to use, it can be shown to be mathematically equivalent to a somewhat more tractable formula:[2]

(4.8) $$S_t = (1 - f - g)^t \left(S_0 - \frac{gL}{f + g}\right) + \frac{gL}{f + g}.$$

Some simple examples indicate that this formula will generate the same results generated by the previous formula. To see this, let us use the formula to derive

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2 This is demonstrated in the appendix. See also Samuel Goldberg, Difference Equations (New York: Wiley, 1958), pp. 63 - 67.

expressions for S_1 and S_2:

$$S_1 = (1 - f - g)^1\left(S_0 - \frac{gL}{f + g}\right) + \frac{gL}{f + g}$$

$$S_1 = (1 - f - g)S_0 + \frac{gL}{f + g}\,[1 - (1 - f - g)]$$

$$S_1 = (1 - f - g)S_0 + \frac{gL}{f + g}\,(f + g)$$

$$S_1 = (1 - f - g)S_0 + gL$$

which is the result that was obtained before. Similarly,

$$S_2 = (1 - f - g)^2\left(S_0 - \frac{gL}{f + g}\right) + \frac{gL}{f + g}$$

$$S_2 = (1 - f - g)^2 S_0 + \frac{gL}{f + g}\,[1 - (1 - f - g)^2]$$

$$S_2 = (1 - f - g)^2 S_0 + \frac{gL}{f + g}\,(2f + 2g - f^2 - g^2 - 2fg)$$

$$S_2 = (1 - f - g)^2 S_0 + \frac{gL}{f + g}\,(f + g)\,(2 - f - g)$$

$$S_2 = (1 - f - g)^2 S_0 + gL(2 - f - g),$$

which was also obtained before (4.3).

1.5 A Numerical Example

A numerical example may help to illustrate the use of the formula to predict support levels. Suppose that for some presidential term parameters are estimated and the resulting model turns out to be:

(5.1) $\Delta S_t = -0.2S_t + 0.3(0.8 - S_t)$.

Suppose, further, that we know that the president started his term with a support level of 0.7 (i.e., $S_0 = 0.7$). We can generate the levels of subsequent support predicted by the model by using the recursive formula:

$$(5.2) \qquad S_{t+1} = (1 - f - g)S_t + gL$$

or by using the general formula:

$$(5.3) \qquad S_t = (1 - f - g)^t \left(S_0 - \frac{gL}{f + g}\right) + \frac{gL}{f + g} .$$

Let us start with the first formula. Then we have

$$S_1 = (1 - 0.2 - 0.3)(0.7) + (0.3)(0.8)$$

$$S_1 = (0.5)(0.7) + 0.24$$

$$S_1 = 0.35 + 0.24 = 0.59.$$

The next time point is computed by

$$S_2 = (1 - 0.2 - 0.3)(0.59) + 0.24$$

$$S_2 = 0.295 + 0.24 = 0.535.$$

If we use the second formula, we have

$$S_1 = (1 - 0.2 - 0.3)^1 \left(0.7 - \frac{0.24}{0.5}\right) + \frac{0.24}{0.5}$$

$$S_1 = (0.5)(0.22) + 0.48$$

$$S_1 = 0.11 + 0.48 = 0.59.$$

And,

$$S_2 = (1 - 0.2 - 0.3)^2 (0.7 - 0.48) + 0.48$$

$$S_2 = (0.25)(0.22) + 0.48$$

$$S_2 = 0.055 + 0.48 = 0.535.$$

The same results are obtained. Note further that support for the president is decreasing even though the probability (g) of winning a new adherent is greater than the probability (f) of losing an old supporter. This is often the case. A president may have a better chance of winning new supporters than of losing old supporters and still experience a declining level of support. This result is explained in Unit 300.

Exercise 3

Consider the graph you drew for Exercise 2. Can you tell why support is declining? Why doesn't support decline all the way to 0?

1.6 Conclusion

It is hard to understand why U.S. presidents seem so likely to lose support during their term in office. If presidents don't try to offend the public, why has no recent president managed to gain support during his tenure? And why do many presidents lose a substantial amount of support while in office?

To answer this question, we have developed a simple model of the way support for a president will change over time. This model expresses the change in a president's level of support from one time to the next as a function of the balance between the losses he suffers from previous supporters and the gains he receives from non-supporters:

$$\Delta S_t = -fS_t + g(L - S_t).$$

As was shown above, this model can be used to predict the level of support a president will receive over time, once the loss rate, f, the gain rate, g, the upper limit on support, L, and the initial level of support, S_0, are known. The appropriate formula is

$$S_t = (1 - f - g)^t \left(S_0 - \frac{gL}{f + g}\right) + \frac{gL}{f + g}.$$

Since we have not yet analyzed the model, we cannot draw many conclusions from it. However, even at this stage one important conclusion can be drawn: *a president may have a greater chance of gaining new supporters than of losing old supporters and still*

You saw this happen in Exercise 2, and the phenomenon can be traced to the fact that the quantity $(1-r-g)^t$ in the formula for S_t decreases in size with increasing t no matter which of the probabilities r and g is larger. The only time $(1-r-g)^t$ fails to approach 0 with increasing t is when r and g are both 0 or both 1. Thus a president's lose of support is not necessarily due to the fact that he has pursued policies that alienate many people. The policies he pursues may be quite popular, in the sense that he gains new support at a higher rate than he loses old support.

2 CONCLUSIONS

2.1 The Normal or Equilibrium Level of Support

In the preceding unit, the expression

$$\Delta S_t = -fS_t + g(L - S_t)$$

was presented as a description of changing presidential support levels over the course of a president's term in office. This equation can be used to generate predicted levels of support for any president according to the formula

$$S_t = (1 - f - g)^t \left(S_0 - \frac{gL}{f+g}\right) + \frac{gL}{f+g} .$$

Thus, for given values of the parameters f, g, and L, and a given initial level of support, S_0, we can analyze what happens to presidential support levels over time.

Let us first consider the quantity $(1 - f - g)^t$ as it changes over time. Note, first of all, that if the quantity $(1 - f - g)$ is greater than -1 but less than 1 -- which will occur if the sum of f and g is between 0 and 2 -- then the term $(1 - f - g)^t$ will decrease over time. This occurs because fractions decrease when raised to higher powers. For example,

$$\left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

$$\left(\frac{1}{2}\right)^3 = \frac{1}{8}$$

$$\left(\frac{1}{2}\right)^4 = \frac{1}{16}$$

$$\left(\frac{1}{2}\right)^5 = \frac{1}{32}$$

$$\left(\frac{1}{2}\right)^6 = \frac{1}{64}.$$

Similarly,

$$\left(\frac{2}{3}\right)^2 = \frac{4}{9}$$

$$\left(\frac{2}{3}\right)^3 = \frac{8}{27}$$

$$\left(\frac{2}{3}\right)^4 = \frac{16}{81}$$

$$\left(\frac{2}{3}\right)^5 = \frac{32}{243}.$$

In fact, over a very long time period the term $(1 - f - g)^t$ will eventually approach 0 if $(1 - f - g)$ is between -1 and 1. $\left(\frac{1}{2}\right)^{10}$, for example, is equal to $\frac{1}{1024}$, a number which is slightly less than 0.001, whereas $\left(\frac{1}{2}\right)^{15}$ is equal to $\frac{1}{32,688}$, a number which is about 0.00003.

The term,

$$S_0 - \frac{gL}{f + g}$$

on the other hand, is simply the difference between two constants and as such will remain constant over time. Thus, the entire quantity represented by

$$(1 - f - g)^t\left(S_0 - \frac{gL}{f + g}\right)$$

is the product of one term which decreases over time (so long as $(1 - f - g)$ is between -1 and +1) and another that stays constant over time. This product, then, will always decrease over time. As an illustration, consider the changes in the quantity $\left(\frac{1}{2}\right)^t(0.7)$ over time.

$$\left(\frac{1}{2}\right)^1(0.7) = 0.35$$

$$\left(\frac{1}{2}\right)^2(0.7) = 0.175$$

$$\left(\frac{1}{2}\right)^3 (0.7) = 0.0875$$

$$\left(\frac{1}{2}\right)^6 (0.7) = 0.0109.$$

For this example, not only is the product of these two terms approaching 0; it is approaching it very rapidly. If this model is applied to monthly data, the example above indicates that within six months the product

$$(1 - f - g)^t \left(S_0 - \frac{gL}{f + g}\right)$$

contributes virtually nothing to the level of S_t. Of course, if different values of the parameters and initial support levels are used, the impact of these two terms might not diminish so quickly. But as long as the quantity $(1 - f - g)$ is greater than -1 but less than +1, the product $(1 - f - g)^t \left(S_0 \frac{gL}{f + g}\right)$ will become smaller over time and will eventually approach zero.

This means, though, that eventually the value of S_t will be determined almost exclusively by the value of $\frac{gL}{f + g}$. To see this, note that if

$$S_t = (1 - f - g)^t \left(S_0 - \frac{gL}{f + g}\right) + \frac{gL}{f + g}$$

and if the quantity

$$(1 - f - g)^t \left(S_0 - \frac{gL}{f + g}\right)$$

becomes very small, then the value of S_t will be almost equal to that of $\frac{gL}{f + g}$. Furthermore, since $(1 - f - g)^t$ is getting smaller and smaller over time, S_t is getting closer and closer to the value of $\frac{gL}{f + g}$ throughout the presidential term. We are thus left with a very important conclusion. *If the quantity $(1 - f - g)$ is greater than -1 but less than 1, then the level of support, S_t, will [illegible] over time towards the value of $\frac{gL}{f + g}$ -- a [illegible] which may be thought of as the normal or equilibrium level of support received by a president during his term in office.*

Let us consider this normal level of support more closely. Since L represents the upper limit of potential supporters, or the maximum proportion of the adult population a president could ever hope to have support him, it is certainly not possible for the normal level of support to be above L. In fact, in most cases we would expect the normal level of support to be considerably below L. How far below L the normal level of support will be should depend on the relative sizes of the probability that the president gains new supporters and the probability that the president loses old supporters. The quantity $\frac{gL}{f + g}$ indicates that the normal level of support will be some fraction (given by $\frac{g}{f + g}$) of L. This fraction is determined by the size of the gain rate relative to that of the total change (gain plus loss) rate. It is conceivable, although hardly likely, that this fraction will be equal to 1, and thus the normal level of support will equal the upper limit on support. In most cases, though, presidents will lose some support during their term in office. Hence f will be greater than zero and the normal level of support will be below the upper limit.

The tendency of the level of support to approach a normal level that is determined by the upper limit on support and the relative size of the gain rate (i.e., $\left(\frac{g}{f + g}\right)L$), is limited to the case where the quantity $(1 - f - g)$ is greater than -1 but less than 1. For our purposes, though, this limitation is not very restrictive. Recall that f represents the probability that a supporter will defect in the interim between two periods when support is measured, and g represents the probability that a non-supporter will begin to support the president during the same interval. But probabilities can never be less than zero (since a probability of zero means that there is no chance of something happening) or greater than 1 (since a probability of one means that the event will always occur). Thus, neither f nor g can take on negative values, nor can either take on a value that is greater than one.

This restriction on the value of f and g implies that, in virtually all cases, the quantity $(1 - f - g)$ will be greater than -1 but less than 1. Therefore, in virtually all cases the level of support will, over the course of a presidential term, approach its normal level. To see this, note that if f and g are each greater than zero but less than one, the condition that $(1 - f - g)$ is greater than -1 but less than 1 is automatically fulfilled. For if

$$0 < f < 1$$

and

$$0 < g < 1$$

then

$$0 < f + g < 2$$

and

$$-2 < -f - g < 0$$

which means that

$$-1 < 1 - f - g < 1.$$

Thus, the only cases we need to consider as exceptions to the restriction that $(1 - f - g)$ lies between -1 and 1 are the two extreme cases where either f and g are both zero or f and g are both one. Although neither of these cases is likely to occur in practice, it is instructive to consider what would happen were one or the other to occur. Suppose, first, that f and g were both equal to 0. This would mean that the president never loses any support nor gains any new support. Hence, his level of support should never change. he will always receive whatever support he received initially. And this is exactly what the model predicts would occur. For if f and g are zero, then $(1 - f - g)$ is equal to 1 and

$$S_t = (1)^t\left(S_0 - \frac{gL}{f+g}\right) + \frac{gL}{f+g}$$

and, since $(1)^t$ is always 1,

$$S_t = S_0$$

If, on the other hand, f and g are both 1, the president loses all supporters he had the previous time period, and gains all of the potential supporters he failed to win over the previous time. Unless the proportion of supporters exactly equals the proportion of potential non-supporters, his level of support will always fluctuate as all supporters shift to non-supporters and all potential non-supporters shift to supporters, and the situation swings back again. And this is what the model would predict. For if f and g are both 1, the quantity $(1 - f - g)$ will be equal to -1. The quantity $(1 - f - g)^t$ will thus be equal to -1 if t is an odd number, and 1 if t is even. Initially, then, $S_t = S_0$. At the next time period,

$$S_t = (-1)\left(S_0 - \frac{gL}{f+g}\right) + \frac{gL}{f+g}$$

$$S_1 = 2\frac{gL}{f+g} - S_0$$

which will be followed by a level of support equal to

$$S_2 = (1)\left[S_0 - \frac{gL}{f+g}\right] + \frac{gL}{f+g}, \quad S_2 = S_0$$

and the level of support will continue to fluctuate between these two levels.

We are thus left with the conclusion that in *virtually all circumstances, the level of support received by a president will change over time so as to approach the normal level of support determined by $\frac{gL}{f+g}$. In extreme cases, however, the level of support will stay exactly equal to the initial level of support or will fluctuate between this level and the level of support received in the period immediately following the initial one.*

2.2 Approaches to the Equilibrium Level of Support

The way in which levels of support approach the equilibrium one may vary considerably. Suppose, first of all, that f and g are fairly low; in particular, that

$$(f + g) < 1.$$

This means that

$$g < 1 - f.$$

Since f is the probability that a supporter will defect, $(1 - f)$ is the probability that a supporter will continue to support the president; i.e., $(1 - f)$ is the rate at which support is retained. The condition we are investigating, then, is one in which the president is more likely to retain support than to gain new support. This situation should be fairly stable, in the sense that changes should be relatively slow and smooth and wild fluctuations should not occur.

If $g < 1 - f$, then the quantity $(1 - f - g)$ will be less than 1, but greater than zero (so long, of course, as f and g are not both equal to zero). Since $(1 - f - g)$ is thus a positive fraction, the quantity $(1 - f - g)^t$ will also always be positive, although it will decrease as t gets larger. Consider what happens to the level of support over time:

$$S_t = (1 - f - g)^t \left(S_0 - \frac{gL}{f + g}\right) + \frac{gL}{f + g} .$$

If the initial level of support is above normal, so

$$S_0 > \frac{gL}{f + g} ,$$

then the entire quantity

$$(1 - f - g)^t \left(S_0 - \frac{gL}{f + g}\right)$$

will always be greater than zero. This means that S_t will always be above $\frac{gL}{f + g}$. However, over time the quantity $(1 - f - g)^t$ will decline and so will the product $(1 - f - g)^t \cdot \left(S_0 - \frac{gL}{f + g}\right)$. This product, then, will add less and less to $\frac{gL}{f + g}$. Thus, S_t will always be greater than $\frac{gL}{f + g}$ but over time it will come closer and closer to the value of $\frac{gL}{f + g}$. This situation is depicted in Figure 1.

Figure 2 illustrates the case where $g < 1 - f$ as before, but the initial level of support is below normal. In this case,

$$S_0 < \frac{gL}{f + g}$$

and

$$S_0 - \frac{gL}{f + g} < 0.$$

Since $(1 - f - g)^t$ is still a decreasing positive number as t gets larger, but $\left(S_0 - \frac{gL}{f + g}\right)$ is now negative, the product of the two will always be

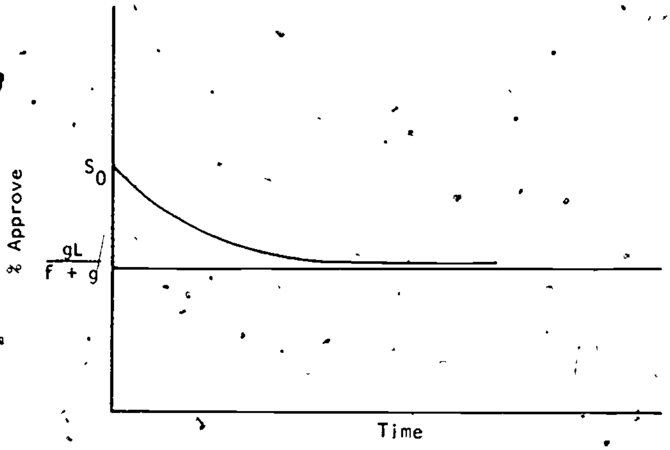


Figure 1. Change in Presidential Support Over Time.
Conditions: Initial support is above normal
$0 < 1 - f - g < 1.$

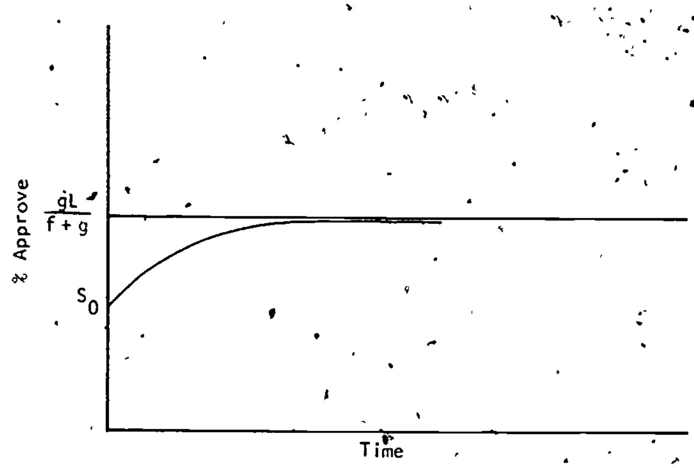


Figure 2. Change in Presidential Support Over Time.
Conditions: Initial support is below normal
$0 < 1 - f - g < 1.$

negative. S_t, therefore, will always be less than $\frac{gL}{f+g}$. Over time, $(1 - f - g)^t$ will become smaller and smaller and therefore a smaller number will be subtracted from $\frac{gL}{f+g}$. Thus S_t will increase over time and eventually approach its normal level.

So long as the retention rate is greater than the gain rate, then, change in the level of support will not exhibit severe fluctuations. Under these conditions, if the initial level of support is above normal, support will steadily decrease and approach the normal level. If initial support is below normal, support will increase, approaching the normal level.

On the other hand, if the gain rate is greater than the retention rate, i.e.,

$$g > 1 - f.$$

then the quantity $(1 - f - g)$ will be less than zero (but still greater than -1 so long as f and g are not both equal to 1). Thus $(1 - f - g)$ will be a negative fraction. But this means that $(1 - f - g)^t$ will be positive if t is an even number and negative if t is an odd number. Thus, if initial support is above normal, S_t will be above $\frac{gL}{f+g}$ if t is even, and below $\frac{gL}{f+g}$ if t is odd. Of course, since the size, or absolute value, of $(1 - f - g)^t$ will decrease over time, the entire quantity $(1 - f - g)^t\left[S_o - \frac{gL}{f+g}\right]$ will contribute less and less additional support to $\frac{gL}{f+g}$ at the even numbered time periods, and detract less and less support from $\frac{gL}{f+g}$ at the odd numbered time periods. The resulting trend in S_t is depicted in Figure 3. If initial support is below normal, the resulting trend will be the same except that now S_t will be below $\frac{gL}{f+g}$ if t is even, and above it if t is odd. In both cases, though, *a negative fraction value for (1 - f - g) produces oscillations that decrease over time.*

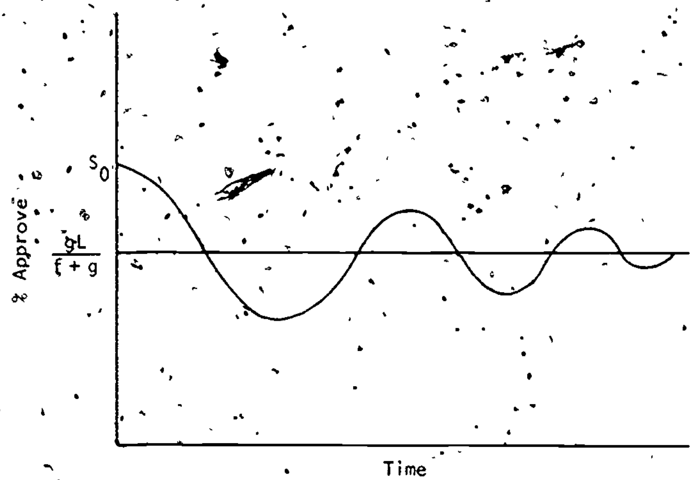
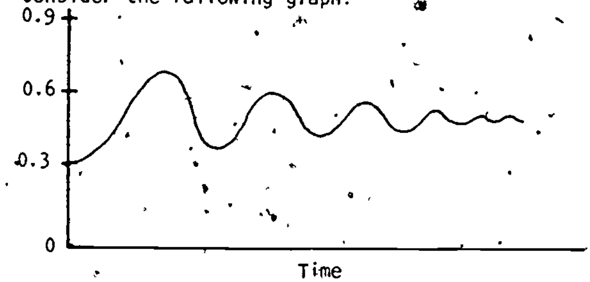


Figure 3. Changes in Presidential Support Over Time.
Conditions: Initial support is above normal
$0 < 1 - f - g < 0.$

This result is not surprising in view of the fact that it occurs when the gain rate is higher than the retention rate. This condition means that either the gain rate is very high or the loss rate is very high. In either case the situation will be in flux: a lot of people are changing their opinions. If f and g are both high, people are changing opinions very frequently. This should result in a considerable amount of fluctuation in the level of support. But even if one of these parameters is high, while the other is not, flucutution should result. For example, if there is a high loss rate and a moderate gain rate, support will decrease as supporters defect in large numbers. But then the pool of potential supporters increases drastically and even a moderate gain rate will result in a net gain of support. The high loss rate means that such gains are not likely to be retained, and the situation can continue in the same vein until it eventually stabilizes.

Exercise 1.

Consider the following graph:



a) What is the normal of equilibrium level of support?

b) What is the initial level of support?

c) What restrictions on the values of the parameters are necessary to produce this graph?

We are now in a position where we can understand how, even in a democratic political setting, many presidents manage to lose support during the course of their presidential term. If campaigns generate unusually high levels of support, as many people have suggested they do, then a trend of decreasing support will be manifest so long as a president retains support at a higher rate than he gains new adherents. But this condition is likely to be realized in the U.S., where party affiliations are quite stable and the level of interest in politics is fairly low. Under these conditions, people are not likely to follow politics very closely and hence many people will tend to retain their impressions of the president once they have initially formed them. Thus the pattern of support represented in Figure 1 is probably typical for the U.S.

This does not mean that this pattern need always occur. It is conceivable, although probably not very likely, that a highly controversial president would attract supporters and alienate followers quickly enough to generate a pattern of support similar to that of Figure 3. It is not likely that such risky candidates would pass the nomination process, but under certain circumstances they might. It is also possible that a president would be so popular once in office that he would attract numerous new supporters throughout his term and actually increase his support level. This situation might occur if the president were widely viewed as the best of two evils at the time of the election but subsequently managed to become quite popular. There is nothing inevitable about the pattern of decreasing presidential support, and in fact it does not always occur nor does it occur to the same degree. During his first term in office, Eisenhower managed to retain an exceptionally high level of support. But if the mobilization of support is unusually high during campaign periods, as is quite likely to occur, and if people do not have highly volatile opinions, as is also likely, a decreasing trend in the level of presidential support should be manifested. And, indeed this is what tends to happen.

Exercise 2

The following data represent Eisenhower's popularity at three month intervals during his first term in office (1953 - 1956):

0.71, 0.74, 0.73, 0.65, 0.69, 0.63, 0.68, 0.63, 0.70, 0.69, 0.76, 0.75, 0.76, 0.71, 0.68, 0.75.

a) Graph these data.

b) What parameter values would most nearly approximate this graph?

c) If you have access to a calculator, graph the level of S_t predicted by the model for the parameter values you chose in part b. (Compare this graph with the one you obtained in (a).

2.3 Empirical Analysis

A statistical tool -- linear regression analysis -- can be used to obtain estimates of the gain rate, loss rate, and upper limit on support for each of the presidential terms. This technique allows us to find the equation of a line that best describes the relationship between two variables. Suppose, for example, the data presented in Table 1 and graphed in Figure 4 were available for five people. The regression line, $y = mx + b$, is determined by choosing values for the coefficients m and b that minimize the sum of the squared differences between actual income and the income predicted by the regression equation for each educational level.[1] The availability of computer programs that calculate m and b

TABLE 1

Income and Education Levels

Years in School	Annual Income
8	$ 7,000
16	18,000
17	25,000
12	12,000
20	20,000

[1] The formulas that determine m and b are

$$m(\Sigma x_i) + na = \Sigma y_i$$

$$m(\Sigma x_i^2) + b(\Sigma x_i) = \Sigma x_i y_i,$$

where n is the number of data points (x_i, y_i), and all the sums run from $i = 1$ to $i = n$. One first finds the sums, then solves the resulting equations for m and b. It can be shown by calculus techniques that the values of m and b found by this method minimize the sum

$$\sum(y_i - mx_i - b)^2$$

of the squares of the vertical distances between the line $y = mx + b$ and the data points (x_i, y_i).

makes it easy for anyone with access to a computer or programmable calculator to obtain the regression line for any set of data on two variables.

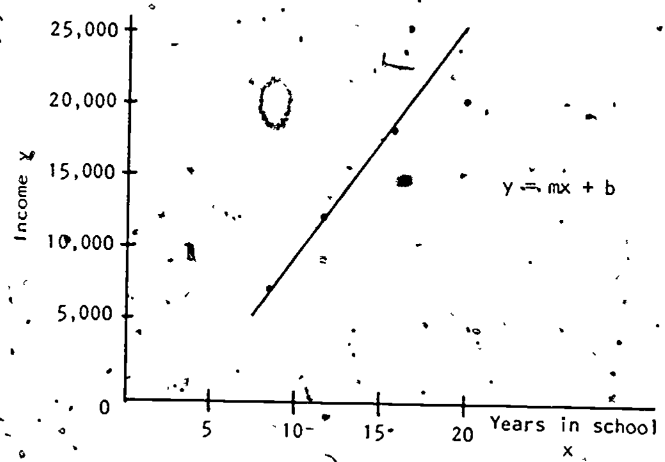


Figure 4. A regression line y = mx + b fitted to the data from Table 1.

This technique helps us to estimate the gain rate, loss rate, and upper limit on support, because the equation

(3.1) $\Delta S_t = -fS_t + g(L-S_t)$,

which describes changes in presidential support, can be rewritten in the form

(3.2) $y = mx + b$,

as we shall now see. Since

$$\Delta S_t = S_{t+1} - S_t,$$

Equation (3.1) is equivalent to the equation

(3.3) $S_{t+1} - S_t = -fS_t + gL - gS_t$.

But, then,

(3.4) $S_{t+1} = S_t - fS_t - gS_t + gL$.

When we factor out an S_t on the right, we obtain

(3.5) $S_{t+1} = (1 - f - g)S_t + gL.$

We may then make the following substitutions:

(3.6) $y = S_{t+1}$

(3.7) $x = S_t$

(3.8) $b = gL$

(3.9) $m = 1 - f - g,$

to convert Equation (3.5) into the basic regression equation,

$$y = mx + b.$$

This means that we can use regression analysis to get estimates for m and b for a given set of data on presidential support levels by using Equation (3.1) with the substitutions given in Equations (3.6)-(3.9). Once the values of a and b are known, Equations (3.8) and (3.9) provide information about the values of f, g and L.

Unfortunately, these equations do not provide enough information to get unique estimates of all three parameters. However, if the value of one of these parameters is known, Equations (3.8) and (3.9) will give us values for the other two. Suppose, for example, that we somehow know the value of L. Then, from Equation (3.8),

(3.10) $g = b/L$

and thus the value of g can be calculated. And, from Equation (3.9),

(3.11) $f = 1 - g - m,$

so the value of f can be calculated.

Of course, in order to use Equations (3.10) and (3.11), we need some way of finding the value of L.

Fortunately, though, we have some knowledge about the upper limit of support a president may expect to receive. Table 2 in Unit 299, reproduced here, indicates that no recent U.S. president has managed to win the support of more than 87% of the population. Furthermore, since Truman, the maximum amount of support received by presidents has declined. Thus we know the maximum support received by each recent president, the maximum support any recent president has received, and the trend in maximum support levels. This information can be used to obtain a fairly reasonable estimate of the upper limit on support for any president.

TABLE 2

The Range of Presidential Popularity

President	Range of Percent Approval Received
Ford	37% - 71%
Nixon	24% - 68%
Johnson	35% - 80%
Kennedy	57% - 83%
Truman	23% - 87%

Suppose, for example, we wish to estimate the value of L for Nixon. At one point Nixon was supported by 68% of the population (his highest level of support) and hence his upper limit of potential supporters must have been at least 68%. On the other hand, his limit was probably below the 87% high reached by Truman, since the maximum support of presidents has declined in more recent years. Thus, although we do not know the value of L for the Nixon years, it is plausible to assume that this parameter was between 0.68 and 0.87. One estimate of L is simply the midpoint of this interval. Since we don't know where L is in the interval between 0.68 and 0.87 our best guess is that it is right in between the two extremes. Thus, for Nixon,

$$(3.12) \qquad L = \frac{0.68 + 0.87}{2} = 0.775.$$

Similar techniques can be used to obtain estimates of L for other presidents.

In summary, obtaining empirical estimates for the parameters of Equation (3.1) requires two steps: 1) Regress the level of support received by a president (S_{t+1}) on its previous level (S_t) to obtain values for a and b, and 2) use estimates of a and b, in conjunction with Equations (3.10), (3.11) and a reasonable estimate of the value of L to find the values of f and g.

Exercise 3 (Requires access to a computer or programmable calculator.)

a) Use the data on Nixon's popularity in Table 1 of Unit 299, reproduced here, to obtain estimates of the regression of Nixon's support on his previous level of support. Then use the regression estimates to obtain values of f, g, and L for Nixon's second term.

Trend in Nixon's Popularity, 1973-1974

	Approve	Disapprove
January, 1973	68%	25%
February	65	25
March	59	32
April	48	40
May	44	45
June	45	45
July	40	49
August	38	54
September	32	59
October	27	60
November	27	63
December	29	60
January, 1974	26	64
February	25	64
March	26	65
April	26	65
May	28	61
June	26	61

Source: Gallup Opinion Index

b) Graph the actual values of Nixon's support over time (as was done for other presidents in Figure 1 in Unit 299) and the levels of support predicted by the model. How close are the two graphs? Can you think of any reasons for some of the discrepancies?

2.4 The Impact of Unusual Events

The model of presidential popularity that has been developed here is very simple. As was earlier noted, actual loss and gain rates probably do not remain constant throughout the course of a presidential term, but vary somewhat as conditions in the country change. Thus, changes in support for a president will probably not follow the smooth pattern predicted by the model, but will deviate to some degree from these patterns. Nevertheless, if changes in the loss and gain rates are relatively small, as they probably often are, the model will work reasonably well in predicting general tendencies in the change of support levels.

However, it is possible for an unusual event to occur that temporarily changes the gain or loss rate considerably. A president, for example, may make an unpopular speech, pull his dog's ears, or make some other mistake that temporarily incurs the wrath of the public. Or, the president may win a tax rebate, announce a major diplomatic victory, or perform some other feat that temporarily wins an unusually high level of support.

The key word here is temporarily. If the president does not suffer any permanent victory or defeat as a result of these actions, we would expect that his level of support would tend to return to the level of support he received prior to the occurrence of the unusual event. On the other hand, since it might take some time for the impact of the event to wear off, the effect of the event may not be negligible in a four year presidential term.

The model of presidential popularity can be used to analyze the effects of such events. If the victory or defeat does not permanently affect the president's image, we may suppose that all parameters -- f, g, and L -- will be the same after the event as they were before the event. The occurrence of the event, then, temporarily displaces the level of support received by the president, but does not affect the probability that the president gains new adherents or loses old supporters, nor does it affect the size of the potential pool of supporters beyond the time when the event occurred.

If this is true, then since f, g, and L are unchanged, presidential support should continue to track to the normal level determined by the value $\frac{gL}{f+g}$. However, it will start tracking to this level from a new place. And since the model predicts that future levels of support depend on current ones, i.e.,

$$S_{t+1} = (1 - f - g)S_t + gL,$$

the unusual event will have an impact on all subsequent levels of support. The effect of the displacement of support, then, is to set a new initial condition. Support will continue to track to the same normal or equilibrium level, but it will do so from a different starting point. An illustration of this phenomenon is depicted in Figure 5.

If the displacement of support following an unusual event is not very large, then unusual events, like small variations in the gain and loss rates, should lead actual changes in presidential support

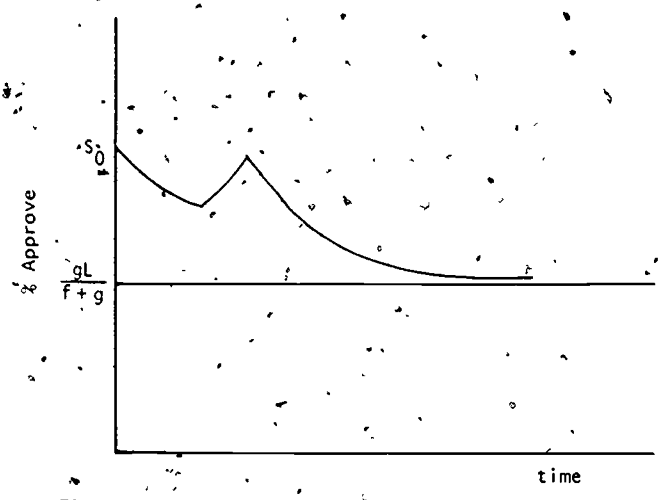
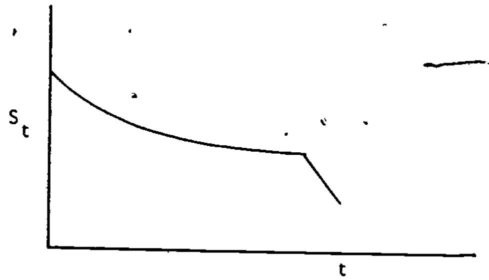


Figure 5. Temporary Displacement in the Level of Support

to exhibit a less smooth pattern than the model predicts, but should not cause actual popularity changes to deviate from the overall pattern predicted by the model. However, if the displacement is very large, then even though support will continue to track to the old normal level, it will do so from such a different starting point that, over the course of a four-year presidential term, actual and predicted changes of support may be quite different. The model, then, would be inadequate to deal with cases where either unusual events lead to huge temporary displacements of support or with cases where gain and loss rates exhibit large amounts of fluctuation during the course of a presidential term.

Exercise 4

Suppose a temporary displacement occurs that does not permanently change any of the parameter values as follows:



Graph the behavior of S_t predicted by the model following the temporary displacement.

2.5 Conclusion

The model of presidential popularity developed here is quite simple: It treats changes in the level of support received by presidents as a function of the balance between the average gains and the average losses of support for a president during his term in office. The model ignores all of the variations in support that stem from short term changes in political, economic, and social conditions, and thus cannot hope to capture all of the variations in presidential support levels that actually occur. Nevertheless, it is powerful enough to capture longer trends in support levels and is helpful in explaining why presidents tend to lose support during their terms in office. In fact, in many cases where the parameters of the model are estimated from survey data, the trend predicted by the model comes surprisingly close to the actual levels of support received by a president over time.

The model is also quite helpful in suggesting conditions under which different patterns of support might emerge. Although many of these conditions are likely to be rare in the context of American presidential politics, they may be realized far more frequently in different contexts. For example, if the model were used to study support for the Chief Executive in some of the countries with multi-party systems, the results would probably be quite different. Then, too, the model might be applied to changing public opinion concerning issues other than presidential popularity. Public opinion on the abortion issue, for example, or on civil rights issues, would in all likelihood follow a different pattern from the one predominant in presidential support.

This is not to suggest that this model can or should be used to study changes in public opinion concerning all issues. There are some issues for which short term forces not only predominate in determining the level of support but are also the most interesting aspects of the analysis of support. Thus, we would be primarily interested in factors that affect the loss and gain rates at any particular time, and would not wish to use a model that assumes constant loss and gain rates. But there are also many issues for which such short term changes are not predominant and thus the assumption of constant loss and gain rates provide a fairly reasonable approximation to reality. For such issues, this model provides a helpful tool in analyzing long term trends in support and the conditions that generate these trends.

3. ANSWERS TO EXERCISES

Unit 299

Exercise 1

a) S_2

b) S_8

c) ΔS_5, or $S_t - S_5$

Exercise 2

$S_1 = 0.76$

$S_2 = 0.73$

$S_3 = 0.70$

$S_4 = 0.68$

$S_5 = 0.67$

$S_6 = 0.65$

$S_7 = 0.64$

$S_8 = 0.63$

$S_9 = 0.63$

$S_{10} = 0.62$

Support is declining, but it is declining at a decreasing rate.

Exercise 3

The model is: $S_{t+1} = 0.8S_t + 0.12$. That is, to calculate the value of S at any time, we take 80% of its previous value and add the quantity 0.12 to the result. Since support starts out high ($S_0 = 0.8$) the quantity 0.12 does not compensate for the loss that occurs as a result of taking only 80% of the previous value. For example, 80% of 0.8 is 0.64, which is 0.16 below 0.8. Thus S_t declines over time. But as S_t declines, 80% of the value of S_t is a smaller number, and so the quantity 0.12 makes up a larger proportion of the gap. Thus 80% of 0.76 is 0.608, which is only 0.152 below 0.76. S_t, in short, declines, but it declines less and less as S_t gets smaller. Furthermore, S_t cannot decline all the way to 0 because the quantity 0.12 always makes up for at least part of the loss that occurs as the result of taking only 80% of the previous value. If S_t could get small enough, 0.12 would more than compensate for this loss. For example, if $S_t = 0.5$, then 80% of 0.5 is 0.4, which is a loss of only 0.1. S_{t+1} would then be 0.52.

Unit 300

1. a) The normal or equilibrium level of support is about 0.5.

   b) The initial level of support is 0.3.

   c) $1-f < g < 1, \quad 0 < f < 1.$

2. Eisenhower's support fluctuates between about 0.63 and 0.76, but it does not do so with any prominent regularity. One solution, therefore, would be to suppose fluctuations in the data occur as the result of minor changes in the gain or loss rate or as the result of temporary displacements. On this solution, Eisenhower's support would be treated as a constant popularity rate (with minor fluctuations due to factors not incorporated in the model); thus f and g are both 0 (or close to 0) and S_0 is about 0.7.

3. $b = 0.034219$

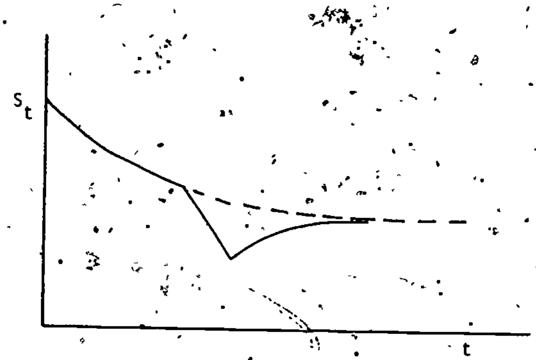
   $m = 0.846596$

   $L = 0.775$

   $g = 0.04$

   $f = 0.11$

4. a.



APPENDIX

DERIVATION OF THE GENERAL SOLUTION

It is not difficult to show that Equation (1.8) of Section 1.4 can be derived from Equation (1.7). However, to do this requires a preliminary mathematical result. This result allows us to simplify the sum of the first n consecutive powers of any number r, as follows:

$$1 + r + r^2 + r^3 + \ldots + r^{n-1} = \frac{1 - r^n}{1 - r}. \tag{A.1}$$

(The first power in the list is $r^0 = 1$.) To see why this formula holds, let the sum of the left-hand side of Equation (A.1) be denoted by S_n. Thus,

$$S_n = 1 + r + r^2 + r^3 + \ldots + r^{n-1}. \tag{A.2}$$

If both sides of Equation (A.2) are multiplied by the number r we have

$$rS_n = r + r^2 + r^3 + r^4 + \ldots + r^n. \tag{A.3}$$

Then, if Equation (A.3) is subtracted from Equation (A.2), most of the terms on the right-hand side will drop out, leaving

$$S_n - rS_n = 1 - r^n. \tag{A.4}$$

To demonstrate the validity of Equation (A.1) all we need to do is to factor S_n out of the left-hand side of Equation (A.4) and then divide both sides of the resulting equation by (1 - r). When we do so we find that

$$S_n = \frac{1 - r^n}{1 - r}. \tag{A.5}$$

Since Equation (A.2) states that S_n is equal to the sum in the left-hand side of Equation (A.1), the result in Equation (A.5) proves that Equation (A.1) is true.

Since r can be equal to any number, we may set r to be the number (1-f-g). Thus Equation (4.7), which states that

$$S_t = (1-f-g)^t S_0 + gL[1+(1-f-g) + (1-f-g)^2 + \ldots + (1-f-g)^{t-1}],$$

may be restated as

$$\text{(A.6)} \qquad S_t = r^t S_0 + gL(1 + r + r^2 + \ldots + r^{t-1}).$$

The quantity in brackets on the right-hand side of (A.6) is a sum like the one in Equation (A.1).

Using the result obtained in Equation (A.1), we have

$$\text{(A.7)} \qquad 1 + r^2 + \ldots + r^{t-1} = \frac{1 - r^t}{1 - r}.$$

Thus Equation (A.6) becomes

$$\text{(A.8)} \qquad S_t = r^t S_0 + gL\left(\frac{1 - r^t}{1 - r}\right).$$

If we substitute (1-f-g) back in place of r, we can obtain Equation (A.9):

$$\text{(A.9)} \qquad S_t = (1-f-g)^t S_0 + gL\left(\frac{1 - (1-f-g)^t}{1 - (1-f-g)}\right).$$

Or, by simplification;

$$\text{(A.10)} \qquad S_t = (1-f-g)^t S_0 + gL\left(\frac{1 - (1-f-g)^t}{f+g}\right).$$

After rearranging terms,

$$\text{(A.11)} \qquad S_t = (1-f-g)^t S_0 + \frac{gL}{f+g} - \frac{gL}{f+g}(1-f-g)^t.$$

Finally,

$$\text{(A.12)} \qquad S_t = (1-f-g)^t\left(S_0 - \frac{gL}{f+g}\right) + \frac{gL}{f+g}$$

which is precisely Equation (4.8).

This unit was presented in preliminary form at the Shambaugh Conference on Mathematics and Political Science Instruction held December, 1977 at the University of Iowa. The Shambaugh fund was established in memory of Benjamin F. Shambaugh who was the first and for forty years served as the chairman of the Department of Political Science at the University of Iowa. The funds bequeathed in his memory have permitted the department to sponsor a series of lectures and conferences on research and instructional topics. The Project would like to thank participants in the Shambaugh Conference for their reviews, and all others who assisted in the production of this unit.

This unit was field-tested and/or student reviewed in preliminary form by: Jonathan Choate, The Groton School, Groton, Massachusetts; Bernice Kastner, Montgomery College, Tacoma Park, Maryland; Ann Holley, San Diego Evening College, San Diego, California; Carol Stokes, Danville Jr. College, Danville, Illinois; Joseph McCormack, The Wheatley School, Old Westbury, New York; and J.W. Uebelacker, University of New Haven, New Haven, Connecticut, and has been revised on the basis of data received from these sites.